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THE TEXAS MATHEMATICS TEACHERS' BULLETIN

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of Democracy, and while guided and controlled by virtue, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

Mirabeau B. Lamar

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Edited by

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Associate Professor of Pure Mathematics

and

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AN UNSOLICITED TESTIMONIAL

The announcement in the newspapers that Professor J. M. Bledsoe of Commerce was to speak on "The Outlook of Mathematics in Secondary Education" at the recent meeting of the Texas Section of the Mathematical Association of America in Dallas led to his receiving the following letter, which bears a tribute to mathematics from an unexpected source. It is reprinted by the editors on account of its general interest. Professor Bledsoe's address appears in this issue of the *Bulletin*.

LABORATORY
WILSON N. JONES HOSPITAL

MEDICAL AND DENTAL X RAY EXAMINATIONS
SUPERFICIAL AND DEEP X RAY TREATMENTS

DR. G. E. HENSCHEN
ROENTGENOLOGIST
PHONE 457
SHERMAN, TEXAS

February 10, 1933.

Prof. J. M. Bledsoe
c/o Southern Methodist University
Dallas, Texas

My dear Professor Bledsoe:

I note with pleasure that you are going to speak on: "The Outlook of Mathematics in Secondary Education."

A word from a layman may be of interest. Some 45 years ago I had my first lesson in algebra from my teacher in Stockholm, Sweden. The introduction was as follows: "We are going to take up a difficult subject that is required for graduation. Excepting for those few of you who intend to teach mathematics you will have no use for this subject as long as you live." When I asked my father for assistance, he told me that neither he, his father, nor grandfather ever had had any real knowledge of mathematics, but that they had all learned it by rote because it was required for a college degree. Father said that no one with our family name had ever been able to really understand mathematics.

Time went on. My boy attended high school. As an introduction to algebra, his teacher told him in exactly the same words as my teacher had told me that the higher mathematics were of no value except for those who intended to teach the subject. Is there a world-wide conspiracy against mathematics?

I believe that no one should be allowed to teach mathematics unless he is able to inspire his students with a desire to learn the subject. The highest culture and the ripest scholarship should be required of the teachers of mathematics in our high schools. Surely no one should be allowed to teach this subject unless he is able to show his students the value of the subject. I am a Roentgenologist, where physics and mathematics are very important; my son is a student in chemical engineering at Rice Institute. Neither of us had decided on our future course when we had our introduction to algebra. Had we been inspired with a desire to learn the subject, our future course would have been easier, and we would have been more useful.

Respectfully yours,
(Signed) G. E. HENSCHEN.

THE OUTLOOK OF MATHEMATICS IN SECONDARY EDUCATION*

By J. M. BLEDSOE
Commerce, Texas

When the chairman kindly invited me to take a place on this program, he asked that I designate some topic which I should like to discuss. In the limited time for deciding on the title of a suitable subject, I could think of none more appropriate than the one which appears on the program, *The Outlook of Mathematics in Secondary Education.*

The Status of Mathematics

Before one may determine with any degree of satisfaction what the future may bring in our educational program, and particularly what may be the fate of such a subject as mathematics, it becomes necessary to make a careful analysis of the situation to determine, if possible, its past and present status in relation to the conditions and needs in our program of secondary education, and upon the basis of this analysis, venture a prediction as to just what the prevailing influences and tendencies may produce.

Those who have made any effort to keep in touch with investigations of various individuals and research committees during the past decade or so have come to think pretty much along the same line, so that no matter what subject may be selected or assigned on a mathematics program, one may expect to hear about the same substance presented. This statement is not made entirely as an apology for my attempt to review briefly some of the findings and recommendations of others who have given serious thought to the conditions and needs in our general scheme of education. It is not my desire to have any one feel that what is contained in this paper is intended in the least as even an attempt to suggest any original idea or newly discovered truth. In my humble judgment, it should be sufficient

*Read before the Texas Section, Mathematical Association of America, at Southern Methodist University, February 11, 1933.

reward to gratify the ambition of any honest man or woman who desires to make some needed contribution in the cause of education, or in behalf of our decadent civilization, to simply be able to share some modest part with those of sane and cautious judgment in their efforts to check the tide of conflagration which seems to be viciously determined to sweep into the abyss of destruction everything which has made substantial contribution towards the permanent establishment and progress of our social, industrial, economic, commercial, educational, moral, and spiritual institutions.

Influence of Personal Interest

In undertaking to speak on an occasion of this kind, before a group of students and teachers of mathematics, and especially in attempting to discuss a mathematical subject, one is naturally reminded of the convention of mice at which a resolution for belling the cat was adopted unanimously with no thought of the question of who could or would undertake the task. We may agree, as has been done so often by groups of mathematics teachers, that our subject is the one of supreme importance in the secondary school curriculum, and complacently console ourselves with the idea that its star has gained a permanent place in the constellation of human intelligence, and that no harm can befall it; yet during recent years, we have seen its form emaciated, and have watched its pedestal made tottering and unstable by unfriendly darts from every direction.

With no thought or intention of impugning the motives that actuate the thoughts and actions of others, it is safe to say that it would be fairly easy for me to guess just what I should say on this occasion to gain hearty approval and applause as I speak before a group of mathematics teachers on a mathematical subject. Realizing this natural tendency in human nature, I shall not expect everything which I have in mind to say to receive your hearty approval, nor to be met with universal applause. In my opinion, better results are obtained when all the facts are brought together for a fair and comprehensive examination and study.

Something Radically Wrong

When one listens with an open mind to the general discussion on the status of our educational outlook, the conclusion may be reached that something must be out of harmony. As to what the remedy may be for correcting the ills in our educational system, little unanimity of opinion has been expressed. Every one who feels an interest in the welfare of our institutions of learning also feels impelled to do what he can in helping to solve some of the difficult, and may I say serious, problems that confront us. In my judgment, no one has yet been able to analyze fully and accurately the conditions and needs, nor has the combined judgment of our leaders been able to find any satisfactory plan of solution. Many facts have been found and stated, and numerous suggestions have been made as to what should be done, yet few seem to be willing to follow the advice or apply the formula set forth by any one or any group of those who are honestly seeking a way out of the dilemma.

Probably our entire program of education is in need of revision, and no doubt some of its items should be eliminated. Just which parts are in need of reform, or which should be strengthened or eliminated, it seems difficult to determine. It would be presumptuous in me, therefore, to assume to speak with assurance in offering a solution of all the difficulties that confront us; however, I wish to suggest a few things which, in my opinion, might help to improve our educational structure by an honest attempt to strengthen one of its parts, *viz.*, mathematics.

Our Own Shortcomings

It may be surprising, or even shocking, to the modesty of the highly technical, or what may be termed the cold-blooded pure mathematician, for one, on an occasion of this kind, to venture any suggestion similar to those which I have just stated. If there should be any one present here tonight who feels that no mathematician, or teacher of mathematics, should ever attempt to think or to speak except in terms of complicated mathematical formulae, let me say that I do not intend any personal discourtesy when I suggest that, in my judgment, such an attitude on the part of many of those who have specialized in the field of mathematics, and who should be ever alert in helping to solve some of the vexing problems in our reorganization program, has been largely responsible for the severe criticisms and great losses which the subject of mathematics has been subjected to during the past quarter-century or more.

The onward march of civilization has passed beyond those conditions or stages, whether for better or worse, when it is possible for an individual or a nation to live and to advance in a state of isolation. The same is true of any subject in the curriculum of the modern school which meets the demands of the present age. The subject of mathematics can no more justify its place in the modern course of study on the theory of isolation, or mathematics for its own sake, than can history or biology. It is true that the study of mathematics has certain cultural and disciplinary values to which I shall refer later on, yet it is practically the unanimous agreement of all the leading educational thinkers and writers that the relative value of a subject in any sane program of education for the masses is directly proportional to its degree of correlation with other fields of study in the experiences and applications of everyday living.

Our Duty in the Reform Program

If the students and teachers of mathematics cannot or will not condescend to bow from their lofty thrones of ethereal mystery, and help to simplify and to vitalize the subject on a level with the common people and the everyday surroundings of practical living, then we shall be compelled to submit to the decrees of those who are not only unfamiliar with its facts and principles, but who are unable to realize and to appreciate its beauties and applications. Much of the stinging criticism of mathematics has been deserved, and has had a telling, and may we hope a salutary, effect. Those who should have been its friends and defenders have been its most destructive enemies. Too long and unjustified have been the desire and the effort of the mathematicians to enshroud the subject in a dark curtain of mystery. Instead of bringing it down to earth where the beauties and utility of its applications can be understood and correlated with the simple and ordinary surroundings and experiences of common living, it has been treated too often in such a dry and uninteresting manner as to repel students rather than to attract them.

There is great danger that the teacher of mathematics will take himself too seriously, and will be tempted to apply too literally the words of Euclid, "There is no royal road to geometry." It is much easier to tell the pupil to study than it is to teach him how to study. The words of Euclid are true only in so far as individual effort is required in the acquisition of any worthwhile accomplishment; nevertheless, the duty of the teacher is not half performed when he has told the pupil to study, or even when he has placed the facts and analysis of the problem fully before the pupil. With modern educational facilities and a properly qualified teacher, the high school student of sixteen may master the principles of Euclid in a term of nine months which required many centuries of undirected thought and effort before the time of Euclid.

Essentials of a Real School

It was a matter of unusual interest and gratification to me some weeks ago to read in *The Dallas News* a statement of the Hon. Nat M. Washer, president of the Texas State Board of Education, who said: "Give us well qualified teachers and a sensible course of study, and a good system of education is guaranteed, regardless of expensive buildings and equipment." Mr. Washer's idea of the supremely essential elements necessary in an efficient system of education is just as simple and as sound as the statement of Garfield when he defined a real university as "Mark Hopkins on one end of a log and a zealous student on the other." The one thing absolutely essential in building a good school is a man or a woman whose soul has been set on fire by the love of the task in hand, which love has been developed through a mastery of the subject and a realization of the beauties of its applications.

The most effective teaching that I have ever seen in a physics laboratory was done by Professor L. G. Allen, now of Canyon, Texas, who made with his own hands every piece of apparatus which he used in illustrating and teaching the fundamental principles of the subject. He could perform more skilful experiments with fewer pieces of apparatus than any man I ever knew, and the secret of it all was that he understood and loved his subject. Magnificent buildings, fine equipment, and expensive libraries no more indicate a great institution of learning than costly garments and glittering jewels portray a character of purity and service.

The Need of Good Teaching

Just a few years ago an incident occurred in a Texas college. I relate this incident simply to illustrate the great importance of properly prepared teachers. A certain teacher had been selected and placed in charge of some classes of college preparatory mathematics, algebra and plane geometry. Just after noon one day, this teacher, in company with a number of students who were being instructed in plane geometry, was walking up the stairway to the recitation room. The assignment for the day was a list of those interesting originals in Book III of Wentworth-Smith's Plane Geometry. One of the young men made the following remark relative to the list of problems: "My, don't we have a lot of hard, knotty problems for today?" This very teacher then actually replied: "Yes, they are awfully hard, and I want you to know that I doubt whether it is worth the effort to solve them." It is unnecessary to give you a report on the progress these students made under the type of instruction which they had, or to describe the tragic failure of this teacher who had such an erroneous notion of what it takes to make an efficient teacher. Why this teacher failed in the attempt to teach plane geometry was not so much a lack of knowledge of the subject as the inability to appreciate and love it.

Imagine, if you can, a real mother after spending the long and anxious hours of the night at the bedside of her firstborn. This mother, pushed on by her undying love and thrilled by the hope of recovery of her child, cools its fevered brow and relieves its pains by her tender caresses and ministrations. Then see her, just at the break of dawn, when the brightness of the sun's rays are driving back the dark curtains of the night, and the infant's smile gives assurance of the effectiveness of the mother's care inspired by her ardent hopes and eager prayers, as she turns to those near by, who themselves are ready and anxious to give assistance and cheer and to share with her the joys of the infant's recovery, as she says: "Oh, well, I have worked so hard, I have had to make such a great sacrifice, have been robbed of a good night of sleep and rest, and after all, I doubt if it is worth the effort."

Mistaken Notion of the Need of Education

Let me make a few general statements relative to the type and need of education which should have the serious thought of every leader in the cause of education. If we agree with Dr. Abraham Flexner as to the purpose and function of a college or university, and naturally with the preliminary corollary respecting the secondary school, we shall be forced to agree that it is not possible, desirable, or necessary that every boy and girl be graduated from a standard high school, much less from a college or university. I shall go a little further by saying that it is not necessary for every one to specialize in mathematics, great as I consider the value and importance of the subject to be. It is neither desirable nor necessary, nor is it possible, for every one to become a master of the field of advanced mathematics. It is not absolutely necessary that every person do his major work in any particular subject in order to complete a respectable high school course; nor is it essential that he become proficient in any particular department to earn a standard college degree.

Probably every leading educational thinker and writer would agree with these statements; yet no one may be considered a safe leader who overlooks the fact that a well-rounded education necessarily implies a mastery of at least one department of learning. And a mastery of any worth-while field of learning requires not only mental ability, maturity of thought, and seasoned experience, but also long and persistent efforts. One who attends classes during a period of four years with no other purpose than to be entertained and to mingle with what he terms his "social group," is no more educated than one who attends picture shows or athletic contests for that purpose. Four years in college, or even the possession of a college degree, have come to mean very little. For the past twenty-five years, the pendulum has been swinging towards the elective system, and towards what some call "the enrichment of the curriculum," in both high school and college, until it has been made possible for a boy or girl, or man or woman, to go through high school and then on through college, without gaining more than a smattering knowledge of anything which requires careful thought or industrious effort.

Function of the Secondary School

The Hon. Charles Evans Hughes, speaking on "The Aim in American Education" before the National Education Association in Boston a few years ago, said: "When we consider the true object of education, to give the training which will enable one to make the most—that is, the best—of oneself, we must realize that the foundation should be laid in a few studies of the highest value, in self-discipline, and that there should be supplied every incentive to attain that mental and spiritual culture which connotes, not mere knowledge and skill, but character. This means self-denial, hard work, the inspiration of teachers with vision, and an appreciation of the privileges and obligations of citizenship in democracy." In speaking more directly concerning the type and extent of training needed in the secondary school, Mr. Hughes said: "It means that sort of training which insists, at whatever cost, on the mastery by the student of the subject before him, on accuracy—the lack of which, I regret to say, is now conspicuous in students of all grades—the correct use of our language, and the acquisition of that modicum of information which every one should possess. In the secondary schools (our high schools and academies) it means that we should stop scattering. There is at present a bewildering and unsuccessful attempt at comprehensiveness. It fails of its purpose in giving neither adequate information nor discipline. It asks too much of the student, and too little. I believe that we need a few fundamental, substantial studies which are thoroughly mastered. I am one of those who believe in the classical and mathematical training, and I do not think that we have found any satisfactory substitute for it. The function of the secondary school is not to teach everything but really to teach something, to lay the basis for the subsequent, and more definitely specialized, intellectual endeavor."

Dr. Flexner, in his epoch-making book *Universities,—American, English, German,* gives some timely criticisms on the character of the course of study in the American secondary schools and colleges. Attention is called to the fact that applicants for admission to college may offer as satisfactory evidence of fitness to do college work the fact of completion in high school of courses in typewriting, bookkeeping, cooking, mechanical drawing, blacksmithing, and kindred subjects. Dr. Frederick Winsor, headmaster of the Middlesex Technical High School, Concord, Massachusetts, insists that such subjects cannot fairly be called studies at all, but are at best merely opportunities to acquire skill. It is pointed out that such courses will be accepted by all but a handful of American colleges on an equal basis with courses in science, mathematics, and languages, which require the use of the mind to master them. Dr. Flexner calls attention further to the fact that the colleges which have admitted students so trained, finding it impossible for them to do intellectual college work, have been forced to provide for them college courses similar to these non-intellectual subjects of school study, and suggests that the list of such courses reads like the advertisement of a correspondence school, such as: "the writing of advertising copy," "elementary stenography," "newspaper practice," "principles of home laundry," "food etiquette and hospitality," "gymnastics and dancing for men, including practice in clog dancing."

Mounting Costs in Education

During the past few years we have heard much about the mounting costs of education, and how economies may be effected in our program of education. In my judgment, the problem is being attacked from the wrong end. Instead of driving the herd of swine from the stream above, our leaders are engaged in the useless and futile procedure of dipping up the water in small vessels, in the hope of clearing up the stream by the process of filtering. Why not drive the swine out of the stream, so that the water will clear itself?

A vicious attack is being made from every direction on the colleges, or what is designated as "higher education." The colleges are only trying to meet the demands made upon them by the secondary schools below. The colleges, especially the teacher-training colleges, are forced to enlarge their scope of training in the curricula offered for the simple reason that the high school has multiplied the number of subjects and departments to such an extent that everything imaginable is included in their course of study. In the latest published biennial report available, that for 1929–1931, it is found that in the high school course of study in Texas, there had been forty-eight subjects affiliated by the State Department of Education, to say nothing about the list of unaffiliated subjects.

And the tragic feature of the matter is that the end of this extravagant folly does not seem to be yet in sight, especially if it is left to the same leadership and influence which have brought it about. This situation has been created by the widespread acceptance of the mistaken theory that every child, regardless of his native ability, inclination, or willingness to work, should be permitted to finish a regular high school course, and then admitted to college and in the allotted time of four years, be graduated and awarded the standard bachelor's degree. As a justification of this colossal pyramiding of the number of courses offered in the high school, it is set forth that Sallie hates algebra and geometry, but that she enjoys drawing comic pictures and making chocolate fudge; hence suitable courses should be provided to enable Sallie to complete the course and be graduated in order that she may pass on into college and enjoy the pleasures of her "social group." It is likewise claimed that Johnny cannot learn history or foreign language, but that he gives promise of becoming an expert in the construction of toy balloons or in the fine art of feeding pigs; therefore, a suitable curriculum should be provided for Johnny's special benefit.

No Criticism of Sane Vocational Training

Now, it is not necessary for me to attempt to defend myself against the charge that I want to criticise the so-called vocational departments which have been added to the secondary school program, for every one who knows me is fully aware of the fact that I am one of the most enthusiastic advocates of such training and skill as the vocational subjects can and should be made to offer.

While a lad at home on the farm, I learned how to feed pigs, how to repair and to build fences, how to lay by an ample supply of fuel wood for the winter, how to prepare and properly fertilize the soil for successful gardens and field crops, how to plant and to care for vineyard and orchard, and dozens of other essential things which every boy should and can learn outside the school room, and which will be learned by every boy and girl who is not obsessed too early in life with the idea that soiled hands produced by honest toil are beneath the dignity of a gentleman or a lady. Let me say that I have found great pleasure all my life in practising those things, for learning which ample opportunity was afforded me without imposing any financial burden on the State. And I shall say further that the lessons I learned, which may also be learned by every boy and girl of common sense and energy, have been worth many times more to me than the lessons supposed to be taught in the secondary school to the boy and girl of the Johnny and Sallie type, for nine times out of ten those who must be fed from a silver spoon in this fashion will spend four years in the high school and four in college, and then go out to spend the remainder of their lives securing their meals at a cafeteria and their wearing apparel at a department store, at least as long as their inheritance holds out.

Vocational training, of the solid and substantial type, when properly organized and presented by capable and serious-minded instructors in the right manner, is fine; but it seems to me that the simple fact should be called to the attention of our so-called leaders and politicians that the State of Texas, or any other state in the American Union, is not financially able to maintain a system of secondary schools with an elaborate curriculum of forty-eight or more subjects taught in each of them. It is time also to learn that it would not be desirable or necessary to undertake to maintain such a system, even if we were financially able. Such courses, if properly organized and provided for, are valuable for those students who will profit from such study and who wish to specialize in these fields, but I want to say, without apology, that I am in hearty agreement with Dr. Winson, who states: "Vocational training has absolutely no place in the secondary school, and should be rigorously excluded from it." He further adds: "For every conceivable occupation in life there are already existing agencies for special training at which a boy can take, after he leaves school, courses which provide a far better preparation for the vocation of his choice than the secondary schools can give. The secondary school needs every moment it can find for its own legitimate fields of instruction, and each vocational course thrust into its curriculum robs its students of some valuable part of its training and gives them in return something of no real value."

Delusion of Overcoming Losses

When one notes the gradual and steady falling behind, not only in the school finances of the State but in other departments as well, one is reminded of the merchant who felt that the small losses sustained in each transaction might be overcome by the great volume of business which he was enabled to do. The secondary schools have certainly been doing an enormous volume of business, and the teachers themselves should be able to answer the question of whether the State is keeping up with the losses which are being sustained. There is every legitimate reason why the State should not jeopardize the welfare of its youth by a policy of spiteful penury towards the public schools. That there must and should be a reduction in the costs of the various departments of the government, there can be no question in the minds of informed men and women. While it is a fact that expenditures must be reduced, it would seem more wise to remedy some of the defects rather than to cripple the legitimate and necessary items.

A Reasonable Solution Suggested

You will pardon me for quoting Dr. Winson again, but he has said so many fine things in such an effective way that I want to pass some of them on to you. He says: "In revising the curriculum for our ideal secondary school, therefore, we must consider not four but three fields of activity for which the pupils need preparation,—their civic relations, their family responsibilities, and the use of their free time,—not forgetting that the consideration underlying all is that the pupils must be mentally well trained. The vehicle for this mental training lies convenient to our hand. The old established course of study will do the trick. Language, science, mathematics, and history have stood the test, and must always be the backbone of the curriculum."

The great slogan for curriculum reform has been "you cannot fit round pegs into square holes." This is true only upon the assumption that the shape and size of the peg or hole cannot be changed. The most vigorous advocates of this assumption would be the first to denounce it, for to admit such an idea would completely upset their psychological theory of child development. If it be admitted that round pegs cannot be made to fit square holes, then it is taken to be just as axiomatic that not all children can learn mathematics. Of course it is true that not all children can learn mathematics upon the same type of assumption that we admit the impossibility of fitting round pegs into square holes. We admit that there are some children who cannot learn mathematics, but when we find such children, it may just as truthfully be said that they cannot learn anything else which requires reasoning power and a disposition to work. The advocates of the slogan referred to above have just as erroneously made use of another platitudinous slogan, *viz.*, "equal educational opportunity for all the boys and girls." No one is a stronger believer in equal educational opportunity than I am for every boy and girl in the land in so far as he or she is capable and willing to pay the price in self-denial and persistent efforts necessary to accomplish the task.

During the past twenty-five years the two educational ideals which have been advocated and emphasized above all others are "equal educational opportunity" and "curriculum reform." While the original purpose of each of these was to encourage and to promote advancement in education, yet the unseasoned judgment and wild enthusiasm of selfish agitators have caused both of these ideals to be perverted into instruments of superficiality, retardation, and injury rather than into beacon lights to lead us on to better things in a program of substantial and enduring educational policies.

False Attitude of Superintendents

Another thing which has helped to bring about the disfavor of mathematics is the fact that so many of our public school superintendents have insisted that anybody can teach mathematics, and as a result people who have known little about the subject and cared less have been assigned classes to teach in high school algebra and plane geometry for no other reason than that they had off periods in their teaching schedules. Not long since, I had a somewhat heated argument with one of those superintendents who claim that a teacher of mathematics in a first class affiliated high school does not need to take college courses in mathematics, inasmuch as the college entrance requirements demand credits in high school algebra and plane geometry, and these are the mathematics courses which the high school teacher has to teach.

For many years, superintendents have selected as instructors, and especially as heads of high school departments, for history, English, foreign language, and science, those with college degrees whose majors were in these respective subjects; and too often have the classes in mathematics been placed in charge of these specialists in other fields during their off periods, with little preparation in or appreciation of the subject of mathematics. We commend in the strongest terms as a safe and sound educational policy the selection of specialists to teach history, English, foreign language, and science, but we want to register an earnest protest against the erroneous and indifferent attitude towards the subject of mathematics.

Wrong Influence on College Graduates

That kind of attitude on the part of those in authority in our schools helps to encourage those who go out to teach in the public schools and who have had no college training in mathematics to undertake to teach the subject in our best high schools. One of our own June graduates in 1929, who had completed four years of college work without even enrolling for a single hour of mathematics, wrote me a letter about the last of August of the same year to recommend him for the position of head of the mathematics department in a strong standard high school. In a sense, these college graduates are not to be censured for this erroneous idea, for the administrators have emphasized the idea that not only mathematics, but any of the well organized subjects which require rigorous and systematic effort on the part of the pupils, are unnecessary and should be either eliminated from the curriculum, or at least made elective. How may these modern graduates be expected to know anything about the value of a thorough and comprehensive knowledge of subject matter in the preparation of teachers when the agitation and tendencies have been entirely in the opposite direction?

In many colleges, not a single hour of mathematics is required for either the B.A. or the B.S. degree. Just recently has the State Department of Education decided to reduce the requirements in high school algebra from two years to one, with alternative choices for the student which absolutely rob the graduation requirement of any semblance of adequate training in the subject.

Now I feel that I speak the sentiments of nearly all the best qualified teachers of mathematics in both the public schools and in the colleges of Texas when I say that I am not opposed to any reasonable or needed reform in the mathematics requirements for high school graduation or college entrance. One would be blind indeed who feels that the last word has been said in any subject, most especially in the subject of mathematics. Even some of the college and university professors of mathematics during the past decade or so have been forced to admit, reluctantly of course, that even the subject of mathematics may be so organized and presented as to make it possible for some individuals, formerly considered incapable of comprehending its dark mysteries, actually to grasp the fundamental principles of mathematics, and to become proficient in the practical applications of its formulae.

Recommendation of the National Committee

In my judgment, the recommendation of the National Committee on Mathematical Requirements is sufficiently sane and adequate to meet the situation. The recommendation of the committee is as follows:

> To the end that all pupils in the period of secondary education shall gain early a broad view of the whole field of elementary mathematics, and, in particular, in order to insure contact with this important element in secondary education on the part of the very large number of pupils who, for one reason or another, drop out of school by the end of the ninth year, the National Committee recommends emphatically that the course of study in mathematics during the seventh, eighth, and ninth years contain the fundamental notions of arithmetic, of algebra, of intuitive geometry, of numerical trigonometry, and at least an introduction to demonstrative geometry, and that this body of material be required of all secondary school pupils.

This recommendation presupposes the regular twelve-grade system in the public school course of study, in which thorough and systematic training has been given during the six years of the elementary school, followed by three years of Junior High School mathematics of the general, or correlated, type which will enable the pupils to gain a broader view of the field than the compartment plan, in use in the Texas program, is capable of giving. If the course of study is so organized as to meet the needs and requirements contemplated by the Committee, the students, by the close of the ninth school year will, as experience has shown, be better prepared to continue the more advanced work in the field of mathematics from the standpoints of both comprehension and appreciation of the subject. Until some sane and feasible plan in the content and organization of the materials in mathematics to be given in the secondary schools has been adopted in Texas, I feel that to reduce the time from two years to one in high school algebra will prove not only disastrous to the subject, but a great injury to the pupils. The fact is, I am of the opinion that no greater injury would result if mathematics should be made entirely elective in the secondary course of study, for in effect it amounts to the same thing if the plan which has been suggested is followed, with the alternative choices provided for the pupil.

Many Would Eliminate Mathematics

It may sound startling to some of you for me to state that there are many school administrators who are not only willing, but are anxiously hoping, to see mathematics made entirely elective in the curriculum of secondary education, and some would vote to have it eliminated entirely. As pointed out above, not a single hour of mathematics is prescribed for either the B.S. or the B.A. degree in many colleges throughout the country. Not long since I heard a Texas college president state in a public address that if he could have his way in fixing the requirements for high school graduation he would include no courses except those in social science. I am glad to say that this statement was not made by the president of The East Texas State Teachers College. I am not overdrawing the situation, nor am I attempting to sound a false alarm when I say that, in my judgment, the most serious element which has contributed to the present alarming crisis in our system of education, not only in Texas, but throughout the country, is the visionary, indefinite, and false conception of the true essence and aim of education.

Our ship of education has been allowed to drift too far from its original moorings. So strong has become the desire for "some new thing" in the minds of many of our would-be educational leaders and politicians that actually they would eliminate the word "progress" from the dictionary, and substitute therefor the word "new," or the word "modern." Now it would be the height of traditional folly to hold on to something simply because it is hoary with age, but there is less reason to introduce something new simply because it is strange and untried.

The Meaning of Education

In determining the relative values of subjects in the curriculum of secondary education, it would seem wise that we have some definite idea as to just what education is. Professor Young, of the University of Chicago faculty, says: "The development of the powers of the pupil to think and to do is the ultimate end of education; while the acquisition of facts is rather one of the instrumentalities used in the attempt to develop this power than itself the end of education." Dewey defines education as "a preparation for complete living." It has also been defined as "the systematic and efficient development of the powers of the mind and the body as a result of the conscious and purposeful efforts put forth in the acquisition of that knowledge and skill which make possible the greatest and most perfect accomplishments of which the individual is capable." Many other definitions of education might be given, yet in each one will be found the inevitable and essential idea that education means the development of the powers of the individual to think and to do as a result of cultured thought.

Place of Mathematics in Education

With this conception of the meaning or aim of education, just what part may the subject of mathematics contribute to this end? Mr. Overman tells us in his *Principles and Methods of Teaching Arithmetic*, page 12, that "The schools must prepare the pupils to meet the new and unforeseen situations; knowledge and habit alone will not enable the pupils to do this. Knowledge is necessary but not in itself sufficient. New situations can be successfully met only by exercise of the higher powers of thinking and reasoning. Can the schools do anything to develop these powers? Undoubtedly, yes. They can and they do. Pupils get more or less 'mental discipline' from every subject studied. Arithmetic is no exception, in fact, it affords unusual opportunities for such discipline."

Young says: "Were its backbone of mathematics removed, our material civilization would inevitably collapse." Abraham Lincoln said that a thorough mastery of Euclid, after he had grown discouraged and disgusted with the legal profession, was the turning point in his life by giving him a clear understanding of what it means to substantiate a proof. Alexander Hamilton is said to have read Euclid through once a month during the days of his greatest statesmanship in order that his logical powers might be kept in effective trim for drawing accurate and reliable conclusions. Professor W. C. Bagley, one of the leading psychologists of America, says in his *Educative Process*, page 211: "The writer is convinced that students who come into his classes in psychology after completing thorough courses in higher mathematics do far better work than those who have not had this training. Something has been carried over from one study to the other. It is certainly not the habit of study, nor are the points that mathematics and psychology have in common sufficient to account for this difference."

Dr. Bolton, in his *Principles of Education*, page 671, illustrates the values to be derived from persistent effort as follows: "We do not wish to have the child do things unwillingly. Things should not be done because they are disagreeable. The two are not mutually exclusive. Every one has felt more self-respect many times when he has persisted in pursuing to the finish some task involving disagreeable drudgery. . . . The farmer boy experiences such a feeling when he finishes well the field of corn among the stumps; binds the bundles in the hot harvest sun, ploughs the stony field, or repairs properly the battered fence. So, too, the child in school feels satisfaction and pride when he has a good geography lesson, a perfect spelling list, or a model page of writing, even though the mind would have feasted on marble-playing, chasing butterflies, making rabbit traps, or going swimming."

The Hon. H. H. Kennedy, a member of one of the leading law firms of Chicago, recently said: "It is my judgment that the study of mathematics very materially assists in developing the reasoning faculties of the mind, and the powers of analysis. In advising a young man to enter upon the profession of the law, I would urge him to take up mathematics above all other subjects in the high school, since I believe that the primary essential to qualify a person to successfully practice law is the mental faculty of reasoning and the power of analysis." He further says: "The science of algebra deals with representations of facts and permits the easier handling of them by the mind. A person well versed in algebra is especially qualified to detach himself from the facts of life in such a way as to enable him to reach conclusions free from bias or personal interest."

Mr. G. M. Reynolds, President of the Continental and Commercial National Bank of Chicago, said: "In planning the education of a child of my own I should insist upon algebra and geometry; for while only a few cases might occur in which the business man would make practical use of either, the mental training acquired in gaining a knowledge of algebra and geometry is of very great help in directing modern business, which is constantly becoming more complex by reason of increasing burdens—a condition which calls for the highest degree of preparation that the schools can give." Similar statements were made by fifty-five of the leading doctors, lawyers, merchants, bankers, contractors, ministers, engineers, etc., of Chicago. Of 105 prominent persons in the city of Cincinnati, and 99 outside the city, with reference to the question: "What course of study should be taken by a boy who is entering high school?" 167 answered that they would require mathematics, 18 would require either mathematics or the classics, and 12 would make mathematics elective.

Desirable Contributions of Mathematics

The reason why I have quoted liberally from others is to give you the exact attitude of those who are successfully engaged in life's responsible activities towards the values that a study of mathematics has actually contributed to the pleasures and possibilities of their vocations.

Students who become proficient in the science of mathematics certainly gain the following powers or transfer values:

(a) The power, or ability, to concentrate attention;
(b) A higher degree of self-confidence;
(c) The power and habit of observation;
(d) The power to make orderly arrangements;

(e) The power to discern correlations and differences;
(f) The power of endurance;
(g) The power and habit of concentration;
(h) Skill in methods of attack;
(i) Dexterity of application;
(j) A sympathetic attitude towards hard work and earnest effort;
(k) The power to make use of other facts gained in the kinship of knowledge;
(l) Greater respect and reverence for truth;
(m) Skill and ability to generalize; and many others.

Briefly answering the empty claim which has ever been unsupported by the facts of scientific experiments, that there are no appreciable transfer values in education, let me give you Dr. Rugg's summary of the results of the thirty-five experiments on this point by thirty of the outstanding authorities in experimental education of the world. This summary is as follows:

(a) Showing no transfer (including one doubtful)	5
(b) Slight gain indicates some transfer	17
(c) Clear evidence of considerable transfer	10
(d) Experimenter claims large amount of transfer (unsupported by evidence)	3
(e) Number of experiments	35

It may be noted that most of these thirty-five tests and experiments were with children whose reasoning powers had not been developed very far.

What of the Future?

We might wonder what will be the status of mathematics in the future program of education. This question may be partially answered by examining the tendencies in our institutions of higher learning. That all engineering students need a considerable amount of mathematics goes without question. Mathematics has always been the basic tool of the physicist and the astronomer, and modern theories involve more advanced mathematics than the earlier ones. The equipment of the present-day chemist involves more mathematical training. Students in the chemistry courses in Yale are now required to have the two years of college mathematics given to engineers, and as stated by Professor W. R. Longley of the Yale faculty, a request is being considered from the department of chemistry to give a special third year course in mathematics for their students. Every one is familiar with the great increase in applications of mathematical statistics in the fields of economics, business, education, and the natural sciences. A short time ago a portion of the staff of the Medical School of Johns Hopkins University formed a class and requested a member of the department of mathematics to give them a course in the calculus. And it is my understanding that it has now been made a requirement of all medical students in Johns Hopkins, and other leading universities, to complete at least one full year of calculus.

But however great the values of mathematics may be, and however strong the demands made upon it by other fields of modern industry, administration, and scientific research, there will still remain the unescapable obligation resting upon the student and teacher of mathematics to help in every way possible in bringing the subject into vital and direct contact with the comprehension of those who should know its principles and appreciate the beauties of its applications.

WORD PROBLEMS IN ALGEBRA*

BY ELIZABETH DICE

North Dallas High School, Dallas, Texas

I believe it will seem quite natural to the classroom teachers of this group if I precede my discussion of classroom activities, word problems in algebra, with some announcements, my extra-class activities.

These leaflets which I wish to give to you are invitations to join the National Council of Teachers of Mathematics. *The Mathematics Teacher*, published monthly except in June, July, August, and September, is, as most of you know, the only magazine in America dealing exclusively with the teaching of mathematics in elementary and secondary schools. Membership in the Council for two dollars entitles one to receive the magazine. Also, this leaflet gives the titles of the National Council Yearbooks, seven of them, without which no mathematical library can be complete. The Eighth Yearbook will be ready for distribution the last of February. The address for the magazine and for the yearbooks is Bureau of Publications, Teachers College, 525 West 120th Street, New York City. Reprints of outstanding articles which have been printed in the *Teacher* can be had from the same address for 10c or 15c. For instance, "Whither Algebra," by William Betz, is well worth 15c.

Also, for mathematical libraries, I wish to recommend the new textbooks. Aside from the variety of problems and the many interesting approaches to different problems, many of these new books have historical notes condensed and in usable form. Many of the publishing companies are glad to give a copy to each school, especially if the copy is to be put in the mathematical library. That is intelligent advertising.

The bibliography for my talk this morning is: Thorndike's *Psychology of Algebra*—still well worth rereading

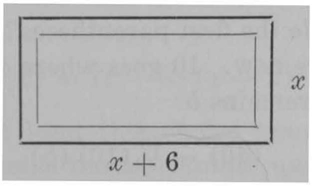
*Given at the Mathematics Section of The Texas State Teachers Association at Fort Worth, Texas, November, 1932.

though published in 1923; *The Mathematics Teacher* for the past five years; the seven yearbooks of the National Council; new textbooks; and my own experience. I shall make no attempt to give specific credit for fragments of quotations which I may use. If these borrowed thoughts so blended with my own that I could not separate them, and if they help any of the beginning teachers here, I know the authors will be glad that I borrowed; and you experienced teachers will, because of that help, be glad to listen to something you already know.

As a basis for the discussion of class activities, I have outlined fourteen approaches to word problems in algebra. Of course, these approaches overlap; some apply to special types of problems; others, while appealing to pupils who think along certain lines, are meaningless to others. Frankly, I admit that I use this or that approach according to the mood I am in. I try to help my pupils to see that each approach is a tool and that the good carpenter has many tools. I am confident that discussing these and other approaches, helping the pupil to analyze, to organize, and to enrich his thinking, and guiding him in pointing out the advantages and disadvantages of a specific approach are far more important than actually solving the problems.

My fourteen approaches (not in the order of importance, because the importance of each depends on the type of problem being considered and on the type of mind doing the considering) are:

1. *The Picture.* A rectangle problem where the length is six feet more than the width lends itself to a picture:



Suppose a path one foot wide is to be put around the rectangle; a picture will help to avoid the danger of calling $x - 1$ instead of $x - 2$ the width of the inner rectangle.

Incidentally, a pupil should not be allowed to think of the area of a path as a series of small rectangles whose corners will treacherously overlap. The area of the path is the area of the outer rectangle minus the area of the inner rectangle.

2. *The Chart.* I like to use a chart in the distance-equals-the-rate-multiplied-by-the-time problem:

	d	r	t
Slow train	$40(t)$	40	t
Fast train	$50(t-2)$	50	$t-2$

A slow train, running at 40 miles per hour, leaving two hours before a fast train, running at 50 miles per hour, is overtaken after t hours. Find t. How natural it is to fill in the chart space by space with 40, 50, t, and $t-2$ as indicated; then, since $d = rt$, the d spaces should have $40(t)$ and $50(t-2)$. Another thought brings one to the equation $50(t-2) = 40(t)$.

3. *The Formula.* There are limitless possibilities in the use of the formula. The area of a triangle is 40 square inches. The altitude is 10 inches. Find the length of the base.

$$(A) = \tfrac{1}{2}(a)(b)$$
$$(\quad) = \tfrac{1}{2}(\quad)(\quad)$$

Vacant parentheses are veritable treasure houses. What did we have inside the first parentheses? A. Then, naturally, 40 goes there now. 10 goes where a used to be. Since b is unknown, it remains b:

$$(40) = \tfrac{1}{2}(10)(b)$$

Of course, formulas should be deduced, solved in terms of each letter, etc.; but I am talking of solving word problems with formulas, not of formulas themselves.

4. *The Statement Approach.* For want of a better name, I have chosen the term the "statement approach." "A cheap kind of gunpowder consists, by weight, of 1 part of charcoal, 1 part of sulphur, and 6 parts of potassium nitrate. How many pounds of each ingredient are there in 160 pounds of gunpowder?"

= pounds of charcoal
= pounds of sulphur
= pounds of potassium nitrate

These statements with the equality signs on the wrong side catch the attention of the child. These equality signs suspended in midair must be rescued. A few skillful questions or a rereading of the problem soon supply letters for the spaces and complete the equation.

5. *The Horizontal Approach.* This approach can be used to advantage in a rectangle-square problem.

"A square contains 4 square feet less than a rectangle whose length is 3 feet more and whose width is 2 feet less than the side of the square. Find the side of the square." Since we are talking of areas, and since we are trying for an equation, we shall indicate the two areas in horizontal form, ready to supply the equality sign when we have balanced the two areas; thus:

$$(x)\,(x) \qquad\qquad (x+3)\,(x-2)$$

Rereading the problem, we see that we shall have the right to put in the equality sign after we have either added 4 to the left group or subtracted 4 from the right group. Four is a number and the right side is the number side, hence

$$(x)\,(x) = (x+3)\,(x-2)-4$$

is preferable. At least this choice temporarily keeps us from having to decide the momentous question of using or of not using the word transposition.

Also, in my horizontal approach I include what I call a rough equation. For instance, John, James, and Bill have

20 marbles, the number owned by each one depending on this or that rigmarole. No matter what the rigmarole, the rough equation is $?_1 + ?_2 + ?_3 = 20$ where each question mark stands for the number of marbles owned by an individual. This rough equation serves as a road map, bringing one back to the main highway, no matter how many times he may detour as he reads conditional and explanatory parts of the problem.

6. *Guessing the Answer.* This, if judiciously done, is a sensible approach. I think I borrowed this idea from "The Teaching of Algebra," the Seventh Yearbook of the National Council of Teachers of Mathematics. "Mary has 3 more dimes than nickels and has a total of $1.65. How many nickels and how many dimes has she?" Guessing the answer is similar to analyzing or supposing that an original problem in geometry is solved. If we guess that Mary has 6 nickels, we have $6 \cdot 5 + 10(6 + 3) = 165$. It is important to use $6 \cdot 5$ and $6 + 3$ rather than 30 and 9 because 6 is only a guess and may need to be changed. In other words, be sure merely to indicate the operations involving the guess. The guess enables the pupil to set up a sensible equation. Why not let x or g or n stand for the guess? Symbols are supposed to stand for something and a guess is something. Erase the 6 and put a letter in its place: $n(5) + 10(n + 3) = 165$.

7. *Reading the Problem.* I encourage the pupils to read, reread, and read again certain phrases of the problem. Time after time I have taken the book (ignoring the tablet and pencil invitingly held before me), slowly read the problem aloud—pausing here and there, probably emphasizing a word or a phrase—thereby encouraging the child to a more logical effort. If it is true that children cannot solve word problems because they cannot read, the thing to do is to read for them. At other times I have them read to me, correcting a wrong emphasis, defining a word, or asking them to tell me what they have read.

8. *Added Details.* Nos. 8 and 9 could be combined with No. 7, but I told you I would give you fourteen. You remember the tea problem? Why not talk of or ask questions

pertaining to grocery stores, and why it is sometimes good business to mix tea? I have idly talked of a reasonable price for tea, whether or not it would be economy to buy cheap tea, the value of a household budget, etc., only to have impatient interruptions from the class insisting that they had solved or believed they could solve the problem without further help. My rambling talk of tea was not rambling at all.

9. *Simplification of Wording.* Instead of saying "The sum of $\frac{1}{3}$ and of $\frac{1}{2}$ a certain number is 10," say "$\frac{1}{3}$ of a certain number plus $\frac{1}{2}$ the same number is 10." Rereading, rewording, adding explanations, asking questions —old approaches, but helpful ones.

10. *Types of Problems.* Many writers have a great deal to say about types of problems, some going so far as to classify the types as uniform motion, mixture, work, age, money, etc. I prefer types based on the operations themselves. The last chapter in The Seventh Yearbook of the National Council of Teachers of Mathematics outlines what I have in mind. For instance: (1) $3x = 6$ where John has 3 *times* as many tops as Jim; (2) $3 + x = 7$, he has 3 *more;* (3) $x + 3x = 12$; (4) $x + (x + 3) = 15$; (5) $5x + 2(x + 3) = ?$; (6) $x + ax + b = c$; (7) $ax + b(s - x) = c$. Such types encourage functional or relational thinking rather than "push-button reflexes."

11. *Making Problems.* One of the best ways to understand the sense of word problems is to make up numerous easy ones. Someone has said: "If you cannot solve a problem, try to solve an easier one like it." Also, making up problems sharpens mentality. I have my Algebra 4 pupils make up problems for Algebra 2. The upper classmen think they are helping me. They are, but when they come to their own more difficult word problems, they find they have also helped themselves. "He who would save his life must lose it." I enjoy hearing the eleventh grade boys and girls discuss the problems they are making for the eighth graders. "Do you suppose those little kids can do this?" or "Is this too hard for them?" or "I'll change this

one. They'll never be able to get it with a fraction in the answer," are some of the scornful yet tolerant questions and comments. Some of the bright eighth graders keenly enjoy asking me to tell my advanced class to make some harder problems for them.

12. *Variety of Solutions.* Nothing gives a child more insight into problem solving than the actual solving of the same problem in several different ways. Why be disturbed if a child lets n equal the larger number? When he sees that $n/3$ equals the smaller number, he will be the one to be disturbed. Guide him in solving the fractional equation mentally or suggest that he can avoid fractions until he has learned how to work with them. I like to encourage the pupils to solve the same problem using one unknown, then two unknowns, fractions, no fractions, feet, inches, formulas, etc. Experimenting? Yes. Wasting time? Perhaps. Learning to think? Pupils can laboriously solve each problem in the text without learning to think, so why not enjoy taking a few problems apart and seeing what they are made of?

13. *Evaluating Answers.* Check answers, approximate answers, and discuss the reasonableness of the answer.

14. *Speed versus Understanding.* No. 14 is the logical outcome of Nos. 12 and 13. Why hurry if the pupils know less after taking your course than they did before? All of you are familiar with the study made by Raleigh Schorling showing not how little mathematics we have taught but how little the children have learned. If we would teach less, they might learn more. Pupils are human enough to like to do what they can do. Let them enjoy using what they learn for a little while before rushing madly on to a new topic. Pupils are human in other ways, too. They like to do some of the talking, they do not like to write all of their thoughts, they dislike drudgery, and they can be taught to know the thrill of succeeding. The arrangement of the new texts gives them an opportunity to talk more, offers suggestions for oral work, and presents numerous

problems involving small numbers. Not all problems, especially the ones with large numbers, need to be finished. Use them for translating exercises, perhaps asking different pupils each day to finish one problem each. There is some point to wading through a maze of numbers if they are to get to tell the class about it. Long assignments, gradual approaches of a cumulative character, and the feeling that a valuable problem cannot be solved in a day—that tomorrow and each succeeding tomorrow will bring a new view, perhaps a clearer one—all of these things point towards independent thinking, learning for learning's sake, and the thrill of succeeding.

Lastly, I would not leave with you the feeling that I think algebra begins and ends with word problems. Word problems merely happen to be my subject for today. Formulas are much more important than word problems. So are certain phases of graphs, and essential skills in algebra. Certainly symbols and equations, the framework of word problems, are more important than the words themselves. Thorndike in his *Psychology of Algebra* says that only one-tenth of the word problems in our textbooks are genuine. He worked out an elaborate point scale, measuring the problems in enough books to make his result meaningful. However, he does admit that the textbooks are improving. Many of our problems are absurd. Why worry over an intricate equation, the result of make-believe conditions, to discover the number of boys in a room? If one really wished to know, he would count them, of course. It is just as absurd to make up word problems for practice in algebraic manipulations. That reminds me of the ancients burning the house to get roast pig!

Pupils are interested in and should be given and should be taught to make or to collect a few genuine problems. Thorndike collected 49. Some pupils are interested in puzzle problems and all pupils should solve a reasonable number of these. One of the 1932 texts has, at the end of each unit, the picture of a clown scratching his head as he thinks of the "brain-teaser" on that page. Some of these problems are of historical value, and if not over-estimated, add to any course in algebra. Present day writers are advocating two other types of problems. First, instead of giving only the material to be used in solving the problem, put in enough extraneous matter to throw "dust in the eyes" of the push-button mind. A pupil has to learn to organize facts, accepting usable ones, rejecting extraneous ones, if he is to cope successfully with various life situations.

Second, why include all the needed facts? Let the pupil not only learn to ascertain what else is needed, but guide him in going to a library in search of it. Some textbook writers insult our intelligence, or at least deprive us of the satisfaction of discovering the index or the tables at the back, with parenthetical expressions such as 1 foot = 12 inches and $\pi = 3.1416$.

Summarizing what I have said: (1) All teachers of mathematics should be members of our National Council and should build up a mathematical library including the *Teacher*, the *Yearbooks*, and new well-thought-out textbooks. (2) No two grown people think alike, so why ask children to think alike or as we do? Give them numerous approaches to word problems and encourage them to make over each one suggested into approaches of their own. Discuss the different approaches with the class, inviting criticisms and suggestions. Lead the pupils consciously to attack the problem from this or that reasonable angle instead of blindly using memorized devices. Remember that relational thinking—not the answer in the answer book or all the problems to the bottom of page so and so—is the goal. (3) Word problems are a part, not the whole, of algebra. Treat them accordingly, always keeping in mind that algebra, the "handmaiden of the sciences," indeed the handmaiden of each product or by-product of the civilization which we enjoy, is far too important a science within itself to be supplanted by any one of its units.

MATHEMATICAL PREPARATION OF A MATHEMATICS TEACHER*

By MARY E. DECHERD
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Perhaps you would like an answer to the question that I have asked myself several times this fall: Why out of the multiplicity of available subjects should I elect to speak with you to-day on the mathematical preparation of a mathematics teacher?

There were, I think, present in my mind rather subconsciously these two reasons.

In the first place, a prominent teacher in one of your state schools told me this summer that he frequently heard it said that there is no necessity of a high school teacher's studying more mathematics than the algebra and geometry that he is to teach. I had thought that such a notion as that had been abandoned in the dim distant past. However, I have recently found in a journal of no mean repute in an article by Dr. W. D. Reeve, of Teachers College, Columbia University, the statement that one "factor retarding progress in the teaching of mathematics is the attitude of certain professors of education in our colleges and universities" who "tell our student teachers that if one is to teach mathematics in the high school, all he needs to know is high school algebra and high school geometry." Surely each one of us here this morning would endorse Dr. Reeve's condemnation of this procedure on the part of education faculties.

Another reason for my choice of subject is the fact brought to our attention by statistics compiled by Supt. B. H. Miller, of the Eagle Pass Public Schools, that only one-third of the teachers of mathematics in our Texas high schools have either majored or minored in mathematics in their university work, and that many of those who did specialize in mathematics in college are teaching other subjects. That this condition is not limited to our Lone Star State is made

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*Read before the Mathematics Section of the Texas State Teachers Association, Ft. Worth, November, 1932.

evident in the same article by Dr. Reeve to which I referred a moment ago. He says: "In spite of the large number of children going to the high school today the need for teachers of mathematics is not so urgent nor teachers so scarce that a superintendent or high school principal is justified in giving a class in algebra to a football coach whose training to teach mathematics is almost nil, or to a second rate teacher who may be good-looking." This entire article of Dr. Reeve's is worthy of your consideration. Its title is "A comparative study of the Teaching of Mathematics in the United States and Germany," and was presented at the annual meeting of the National Council of Teachers of Mathematics at Washington, D. C., in February, 1932. It is printed in the May, 1932, number of the *Mathematics Teacher* and reprints may be obtained at 10 cents each.

On this same subject may I quote also from the Report by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America? This report has for its title "The Reorganization of Mathematics in Secondary Education." Of the constitution of this committee I shall speak later. Just now I desire merely to quote from Part I (General Principles and Recommendations), ch. II (Aims of Mathematical Instruction—General Principles), V (The Training of Teachers), p. 16: "In the meantime everything possible should be done to improve the present situation. One of the most vicious and widespread practices consists in assigning a class in mathematics to a teacher who has had no special training in the subject and whose interests lie elsewhere because in the construction of the time schedule he or she happens to have a vacant period at that time. This is done on the principle, apparently, that 'anybody can teach mathematics' by simply following a textbook and devoting 90 per cent of the time to drill in algebraic manipulation or to the recitation of the memorized demonstration of the theorems in geometry."

Obviously, most of us can have very little to do with the actual placement of teachers of mathematics. I think though that we should feel greatly encouraged to do what we can in the matter. School boards and administrators can not afford to disregard permanently the opinions of teachers and investigators at a point obviously vital to the teaching of mathematics, especially when the leading mathematical organizations, both national and international, are conducting the investigations.

The last statement that I shall cite for you on this point is that of Dr. E. R. Hedrick, professor of mathematics in the University of California at Los Angeles, a mathematician of no mean ability, who nevertheless has been long interested in the teaching of mathematics.

In an address at the banquet of the National Council of Teachers of Mathematics last February in Washington, D. C., he spoke as follows: "I shall not outline here, as I have done elsewhere, the variety of causes that tend toward formalism in our schools. One only seems to me to be particularly germane here. It is the question of the training of our teachers, a subject just now under intensive study by the Federal Bureau of Education, and by the International Commission on the Teaching of Mathematics, of which Dr. David Eugene Smith is the International Chairman. These matters are strongly before me, since I have the honor to be the chairman of the American committee of the International Commission, the other members being Prof. W. D. Reeve, of Teachers College, Columbia, Prof. Eva M. Luse, of Iowa Teachers College, Dr. Ben Frazier, of the Bureau of Education, and Mr. Ben Sueltz, of the State Normal School at Cortland, New York. When the Federal Survey has been completed, and has been digested, as Professor Reeve and some of his students are now doing for my committee, we shall know in detail about the training of teachers in the whole country. Already, however, we know that it is worse than your most pessimistic fears, in instances in which statistics do exist. Whole states exist—in the north—in which one-third of the high school teachers of mathematics who have a college degree, took not one hour of mathematical work in college."

He concludes: "Shall we expect such persons, demonstrably lacking any interest in mathematics themselves to inspire their students to be interested? Shall we expect them to show what mathematics means to the world? Shall we expect through them to escape from improper formalism?" It is unnecessary, I think, to quote further remarks concerning this point, so I shall pass to another phase of the subject.

Before proceeding further, I wish to say a word about both the Committees to which I have referred above. In 1908, the International Congress of Mathematicians met in Rome, Italy. At this time an International Commission on the Teaching of Mathematics was organized with the purpose of getting reports on the teaching of mathematics in the different countries. Many of these reports have been prepared. Professor R. C. Archibald, of Brown University, in a Bulletin of the U. S. Bureau of Education (1918) has given a most interesting and comprehensive resumé of some of these reports together with a report of his own investigations concerning mathematical teaching in the United States.

The other committee to which I referred, The National Committee on Mathematical Requirements, was organized in 1916 under the auspices of The Mathematical Association of America. Its purpose was to coördinate the several reform movements in the teaching of mathematics that were developing in different parts of the United States. Their report was published in 1923 and gives a most detailed and suggestive account of various experiments and investigations made under the direction of the Committee. I shall quote frequently from both reports. Both organizations are at present carrying on further investigations. It seems to me that we may profit from a consideration of various statements in these reports although we may not always entirely agree with them. What is said is certainly far from complimentary to the teachers of mathematics in the United States, but we must remember that the reports are prepared by men themselves teachers of mathematics, well-informed in regard to conditions and conscientiously endeavoring to aid in the advancement of mathematics in the United States.

In each of the following excerpts, I ask your attention to these three points: (1) The opinion set forth concerning the character of teaching of mathematics in the United

States; (2) The comparison between the teaching of mathematics in the United States and in other countries; and (3) The hopeful attitude as to the future of mathematics.

Dr. W. D. Reeve in the article previously referred to writes as follows: "There is great danger, however, in our insularity of mind. Our American teachers need to learn of the salient features of the teaching of mathematics in other great countries of the world just as much as they need to learn of art, literature, politics, and the like. In fact it is as dangerous to foster nationalism in mathematics as it is to be nationalistic in our political views.

"The fact that our country is new and that the high school population has increased so fast in the last thirty years has made it impossible for us to demand as high qualification for teachers as we should like to have had.

"Another significant factor is the fact that our unprecedented industrial and economic growth has resulted in men's leaving the teaching profession to go into other lines of work. The result of this is the domination of women in the classrooms throughout the land. While we may admit the natural superiority of women with children we must not fail to realize that for most of our women teaching is not a permanent profession. What is the mental effect? I think it has led to a lack of interest in the proper preparation and background for teaching mathematics in the schools. As a result we do not have among either men or women teachers enough people of scholarly minds. The mathematical background of many of our teachers is decidedly limited, to say nothing of their deficiencies along other lines. Most of the teachers of mathematics in the secondary schools of Germany are men most of whom have had academic training in mathematics culminating in the Ph.D. degree. Moreover, most of the teachers of mathematics that I saw at work in Germany can teach physics equally well. A great many of our teachers of mathematics would have a hard time to follow the ordinary classroom discussion of such teachers.

He continues: "One would naturally expect to find men teachers in the boys' schools (Knaben Schulen) of Germany, but even in the girls' schools (Mädchen Schulen) one often finds men teaching both mathematics and physics. In fact I had a hard time to find women teaching mathematics at all, but the ones I saw were good."

In the introduction to his bulletin (p. 4) Dr. Archibald makes this statement: "At the present time superintendents, inspectors and principals in many parts of the United States have been forced by public opinion to consider numerous radical changes in methods of secondary school education. If a high minimum standard of preparation were required on the part of a teacher, and the position of the teacher were made such as to attract in sufficient numbers the best talent in the country, other difficulties would disappear. Most countries considered in this bulletin have a far higher standard than we with respect to teachers of mathematics in secondary schools. The degree of this superiority is exhibited throughout the following pages, and some of the chief points are summarized in the last chapter."

On pages 15 and 16 of their report on the Reorganization of Mathematics in Secondary Education, the National Committee makes the rather strong statements: "The greater part of the failure of mathematics is due to poor teaching. Good teachers have in the past succeeded, and will continue to succeed, in achieving highly satisfactory results with the traditional material; poor teachers will not succeed even with the newer and better materials.

"The United States is far behind Europe in the scientific and professional training required of its secondary school teachers (see chap. XIV). The equivalent of two or three years of graduate and professional training in addition to a general college course is the normal requirement for secondary school teachers in most European countries. Moreover, the recognized position of the teacher in the community must be such as to attract men and women of the highest ability into the profession. This means not only higher salaries but smaller classes and more leisure for continued study and professional advancement. It will doubtless require a considerable time before the public can be educated to realize the wisdom of taxing itself sufficiently to bring about the desired result. But if this ideal is continually advanced and supported by sound argument there is every reason to hope that in time the goal may be reached.

"It will be apparent from the study of this report that a successful teacher of mathematics must not only be highly trained in his subject and have genuine enthusiasm for it but must have also peculiar attributes of personality and, above all, insight of a high order into the psychology of the learning process as related to the higher mental activities. Administrators should never lose sight of the fact that while mathematics if properly taught is one of the most important, interesting and valuable subjects of the curriculum, it is also one of the most difficult to teach successfully."

These three excerpts will, I think, give adequately the consensus of opinion concerning the points mentioned above. The conclusions seem to be:

(1) Mathematics is poorly taught in the United States.

(2) In many foreign lands the scholarship of secondary teachers is higher than it is in the United States.

(3) There seems to be reason to believe that, these deficiencies recognized, there can and will be a great improvement in conditions.

It may perhaps be of interest to speak very briefly of the standards for the training of secondary teachers in each of the 17 countries included in the report of the International Commission.

I. In Australia, while the better schools prefer graduates with "honors" in mathematics, it is not necessary that the "teacher of mathematics shall have had special courses in mathematics in the university."

II. In Austria, requirements are high and "it is deplored that teachers of mathematics in the Untergymnasia and Unterrealschule (pupils' ages 10–14) may have had only the scientific training required when mathematics has been taken as a minor." The candidate, when mathematics is a minor, is required "to have knowledge of elementary arithmetic, insight into the structure of the field of real numbers, and into operations with them," elementary geometry and exercise in space perception, accuracy and speed in elements of differential and integral calculus. Mathematics as a major requires familiarity with general arithmetic, foundations of higher algebra and theory of numbers, elementary geometry, synthetic and analytic, differential and integral calculus and its applications to geometry, the elements of the calculus of variations, and foundations of the modern theory of functions and principal results of investigations concerning the foundations of mathematics.

III. In Belgium, the teacher of mathematics in the secondary school must have had a most thorough four years' course in a university including a large number of courses in mathematics and the physical sciences.

IV. In Denmark, the university course for training the teacher for the secondary school requires 8 years. The training in mathematics is extensive and calls also for astronomy and applied mathematics or chemistry with physics.

V. In England, very much the same conditions exist as in the United States.

VI. In Finland, not only is preparation good, but the professor in the secondary school must have a degree and must have majored in mathematics.

VII. In France, the preparation of the teacher in the lycée, the highest type of secondary school in France, is decidedly extensive and is highly specialized. The professional training is almost entirely omitted.

VIII. Denmark, France, Sweden, and Germany have the highest standards for teaching in their secondary schools. Germany pays much attention also to professional training. The teaching force of these secondary schools in Germany contains some of the best mathematical talent in the country.

IX. In Hungary, standards are high.

X. The same remark may be made about teachers of the secondary schools in Italy.

XI. In Japan, teachers in the 8 higher middle schools are nearly all gakushi in mathematics. All gakushi have passed examinations in calculus, differential equations, solid analytics, projective geometry, astronomy and least squares, general physics, general dynamics, and theory of numbers.

XII. In the Netherlands, teachers in secondary schools uniformly hold high degrees, but lack professional training.

XIII. In Roumania, the schools follow largely French standards.

XIV. Russia provides special training for teachers of mathematics for secondary schools.

XV. Standards in Spain are similar to those in Italy.

XVI. As stated before, the schools in Sweden rank as high as any in Europe.

XVII. In Switzerland, scientific standards are high, but professional training is neglected.

XVIII. In the United States, degrees are required in general for the teachers in the best secondary schools, but no special preparation in the subject taught and no professional training are as a general thing demanded.

In other words Australia, England, and the United States are the laggards in the amount of preparation required of their teachers of mathematics in secondary schools, though conditions in England are not as bad as in the United States and Australia.

There are now several questions which I should like to ask you. I wish it were possible for us to have an open discussion of them but since that is impossible, I shall attempt an answer to them myself.

Is the situation in regard to mathematics in our Texas high schools at present satisfactory to you? (If not, what should be changed?) Is it possible that the mathematical preparation of the teachers in the high schools is an influential factor in determining conditions? What can be done about the situation?

As to the first question, I feel confident of your reply. I doubt if any one of us is satisfied with the treatment accorded mathematics today. The number of failures in our classes alone would be disquieting to us. Again many students would escape mathematics altogether if possible, and many more take only what is required of them. Mathematics is being dropped from university and college and even from high school requirements. Do you think the fact that in some cases the teacher of mathematics himself had no enthusiasm for mathematics, no interest in mathematics in his own university career is unavoidably communicated to the student in his classes? Mathematics demands persistence, careful thinking, and insight. Is not this the reason that many students avoid it? What reason have we to think that when these students become teachers they will be good teachers of a subject they avoided in their university work? Sometimes the avoidance was accidental perhaps, but do you not know of many instances in which it was intentional? Many of my former students have said to me: "You are going to tell me that I am not doing right to teach this high school mathematics as I had only Math 1 in the University." If such a person really has the proper attitude to a subject which he is to teach he can now, of course, remedy the defects in his undergraduate work by taking further work in mathematics for his M.A. This is often done and done well. Nor am I forgetful that some teachers who have had very little training are really doing better work than others with excellent training. But these are the very teachers who long for better preparation and will get it if it is at all possible. They realize keenly their handicap; their inability to render the best service of which they might be capable. Again you may say properly that most high school teachers are entirely too busy to study during the session. Yes, that is true, but is not that a condition that should be remedied? It is already being considered by the International Commission.

In the second place, do we not find in all our classes a tendency toward formalism on the part of the students? Formulas themselves are rarely well used; substitution in formulas is generally poorly done. Yet many sciences depend much on the use of mathematical formulas. In studying mathematics an inferior student often prefers to use a formula rather than to think a process through without a formula. E.g., if a student is told that the ratio of similitude between two sides x and a of two $\triangle$s is 2:3 and a is 12, he generally insists on writing $2:3::x:12$, and then $3x = 24$, and $x = 8$ instead of taking $2/3$ of 12 and getting 8 at once. I wonder if such a student was not drilled more on the form than on the underlying principle. In analytics, it takes real strength of character to induce students to understand the formulas that they are using. They have much difficulty seeing that a fundamental geometric method conserves time and effort in the long run as it inducts then into the only general method of deriving equations (in analytics). What wonder that students who see only formulas and formalism in mathematics think lightly of its value. Are we as teachers not taking the very foundations from under our own subject when we allow students to handle any formula before they understand it and to use a formula when a short mental process can be used? Another illustration in point is the joy with which a student inverts the denominator of a complex fraction—a process that he can never explain—instead of using the fundamental process of multiplying numerator and denominator of the complex fraction by the least common multiple of the denominators of the fractions they contain, thus applying the fundamental principle of fractions. The latter process the student sometimes says he does not understand, while he avers that he does understand the former, not realizing that only by this latter process can the former be properly explained.

But why this discussion on formalism? Because there is much justice perhaps in the position of Dr. Hedrick in his article on "What Mathematics Means to the World," where he urges that the reason the value of mathematics is not recognized by most of those who study it in the high schools is the formal way in which it has been taught. "Why is it," he asks, "that the meaning of mathematics is not brought out and brought home to our students?" His answer is "that it is due to the tremendous tendency toward formalism in mathematical teaching." And the cause that he stresses as leading to formalism in teaching is lack of proper preparation on the part of the teacher. He really goes so far as to say that the teacher does not know the truth underlying the formulas which are being taught.

Another argument for adequate mathematical preparation on the part of the secondary teachers of mathematics is a psychological one. If the teacher knows only synthetic geometry and algebra or even has trigonometry in addition, can he have the proper confidence in himself, or will he not often feel an uncertainty and lack of mathematical soundness in himself and in his teaching which will subconsciously affect the character of his work? There is a tendency strong in humanity to respect and even reverence with fear the unknown and the unexplored. A teacher unfamiliar with calculus or other more advanced mathematics will undoubtedly think of it as more or less difficult and perhaps unattainable. Mexico seems much nearer and more vital and interesting to me since my sojourn of a month there this summer. Undoubtedly when calculus and analytic geometry are *terra incognita* to a high school teacher, there is great likelihood that his pupils will have a negative psychological attitude toward higher mathematics.

Another reason for mathematical preparation on the part of the teacher is that only thus can the various applications of mathematics be realized, and hence its far-reaching usefulness appreciated. Many students think that "applied mathematics" is a different subject from "pure mathematics." They are surprised to hear that all mathematics is "pure." It is, I think, unfortunate that various departments in the universities which are using mathematics give their own courses in the mathematics they need. I wonder if this is not because we as teachers of mathematics are also narrow in our sympathies? Mathematicians are to blame for letting Education faculties write the books on the teaching of mathematics. Likewise we who most revere mathematics and call her the queen of the sciences, must familiarize ourselves more thoroughly with the usefulness of mathematics. Sciences will for the most part be retarded in their advancement unless mathematics renders her indispensable aid. A pabulum of high school geometry and algebra and even trigonometry will not give a teacher insight into the applications of mathematics to the sciences. That mathematics has application to engineering and physics and insurance and architecture and chemistry and even to the social sciences and to pedagogy and to music and to many other fields should be full of meaning to any teacher of mathematics. I must admit my surprise last year when a friend of mine writing a textbook on French had to employ a rather expert student of mathematics to work out for her some of the subject matter in her book.

Recently the American Chemical Society held a Colloquium on the need of mathematics in chemistry. Dr. Hedrick is authority for the statement that a committee of the Social Science Research Council has listed mathematics up to and including calculus as a requirement for a social science major. Surely it seems that the foes of mathematics are they of her own household!

One of the strongest arguments for the study of trigonometry, analytics, calculus, etc., for a high school teacher of mathematics is that only thus can the subject matter of algebra and geometry be learned thoroughly. Dr. Dunham Jackson, of the University of Minnesota, has the following to say on this point: "College courses recall the facts and the methods of high school mathematics and give increased familiarity with them, in a setting which shows them in new relations and throws new light on their significance: practically all courses require the constant practice of algebraic technique, trigonometry is an elaboration of the fundamental notion of similar triangles, analytic geometry is a systematic coördination of algebra and geometry, etc., while such courses as college algebra and the theory of equations, or Professor Altshiller Court's college geometry, serve directly to deepen the prospective teacher's experience of high school subjects.

"While elementary geometry goes a long way back into antiquity, and even elementary algebra is largely of ancient or medieval origin, so that there is some basis for the feeling that these subjects are 'cut and dried,' analytic geometry and the calculus, in their modern form at least, are newer than the civilization of what we call the New World, and bring the vitality of mathematics as a living science down to the present day. If one studies college mathematics with the right kind of insight it gives cumulative experience of the notion of limit, which is fundamental in modern mathematics but is left in so unsatisfactory a state in high school, and of the distinction between a proposition and its converse, which is so essential not only in geometry but in all sound thinking."

Lastly, I shall endeavor to answer the question as to what would be adequate mathematical preparation on the part of the high school teacher. I hope you have read the excellent article by Mr. W. J. Edmonston, of Dallas, in the October *Outlook* on "Background of the Mathematics Teacher." In that article he quotes the late Dr. J. W. Young as suggesting for the minimum requirement for a teacher of mathematics the following: trigonometry (surveying desirable), college algebra, plane analytic geometry, differential and integral calculus, theory of equations, and the elements of mechanics, the history of mathematics, and some work in theoretical and practical physics. He further recommends modern synthetic geometry, solid analytic geometry, theory of functions (real and complex) and the theory of numbers. Though changes and substitutions might be made in this list I think in the main it would give general satisfaction.

One further remark: when the student reaches the high school, he has already been in school eight years and the work done in the high school is more or less dependent on the previous performance. Arithmetic has been "finished" and algebra and geometry begun in these eight grades. The question must be asked concerning the preparation of these earlier teachers of mathematics. So far as I can see, it is just as vital that their preparation be sound as for high school teachers. In fact, I believe that high school mathematics is easier to teach than arithmetic. Is Dr. Charles H. Judd, of Chicago, a pessimist or a well informed man? At all events he says, "It is certain that the schools of our day do not know how to teach arithmetic." We have long ago learned that the requirement of a degree is a purely formal matter and may be valueless. Unless the degree represents achievement, it is valueless, of course, and yet it does represent spending some years in an academic atmosphere. More attention should be paid to the content of the degree, of course. President Lowell of Harvard says that graduate schools exist for teachers. Perhaps so, and many teachers have M.A. degrees and many more are getting them. Yet we can not estimate learning by degrees.

The question must work itself out through the years. The world needs mathematics. As one of your state dailies said in an editorial "If one says he cannot learn mathematics, he comes dangerously near saying he cannot think." Kagawa's reform measure in Japanese slums was to teach arithmetic and algebra. President Lowell more than twenty years ago by careful investigation came to the to him unexpected conclusion that the best preparation for a law degree was intensive courses in mathematics. I think it was Dr. F. J. Kelly who calls our attention to the fact that for fifty years the leaders of civilization have been college trained men and women. Surely it is time for us to consider our entire educational system. As a nation we have, I fear, little faith in our political and financial leadership. More tragic still would be loss of confidence in our schools.

If we who are teaching mathematics do not set ourselves resolutely to the task of improvement in the teaching of our own subject, the entire system will suffer. No selfish end should be tolerated. We are not undertaking this task for our own good, but because of the indispensable contributions that mathematics can make to the development of the individual and to the progress of the world. Mathematics has values both mental and moral, both material and spiritual, both temporal and eternal. I make no empty boast when I say that the world is poorer when the importance of mathematics is ignored. Let us not be discouraged by the present trend in educational matters. I long ago began to acknowledge frankly that it takes time and effort to learn mathematics and this fact alone would make it of value. Easy subjects are clamored for in many quarters, but surely this is a passing phase in education. The truth and rightness of mathematics abide. Let us not leave it for the next generation to advance the position of mathematics and put her again in her rightful place. Let us do it ourselves. We can and I believe that we will.

THE BROWN PRIZE EXAMINATION

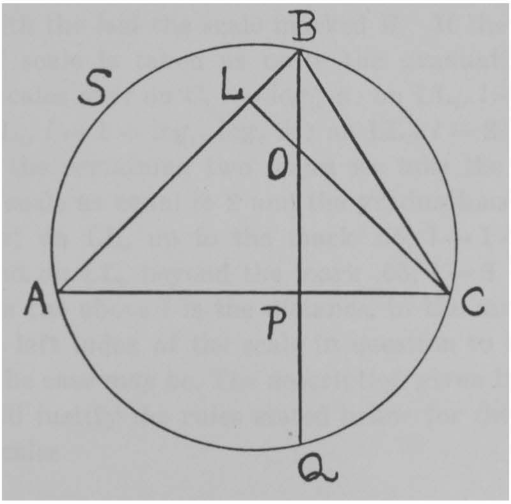
The Brown University Entrance Prize Examination was given this year on October 8 at 2:00 P.M. Sixty-six people came to take the examination. There were 32 papers handed in. The first prize was given to Richard Fleming of San Antonio, the second to Raymond M. Tucker, Jr., of Sherman, and the third was divided between Woodrow Alexander and Pauline Chrisman, both of Austin. The amounts of money for the prizes were as follows:

First Prize	$13.75
Second Prize	9.00
Third Prize	4.50

The number who came for the examination was decidedly gratifying. At the end of the first hour more than one-half of the students still remained. There has never been a year, it seems, in which there was so much interest taken in the examination. The questions were as follows:

1. Two sides of a triangle have the lengths 10 and 12, and the altitude on the first side has the length 8; find the length of the altitude on the second side.

2. Prove that if the triangle ABC is inscribed in the circle S, $OP = PQ$, where O is the intersection of the altitudes BP and CL.



3. The simple interest payable to A is \$120. How much simple interest should be payable to B if B's principal is twice that of A, but the time of B's loan is only two-thirds as long as that of A, and B is charging 4 per cent instead of 5 per cent charged by A?

4. Two places, A and B, are 168 miles apart, and trains leave A for B and B for A at the same time. They pass each other at the end of 1 hour and 52 minutes, and the first reaches B half an hour before the second reaches A. Find the speed of each train.

ON THE USE OF THE SLIDE RULE

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These notes contain some rules intended to simplify the use of the log log scales on the slide rule and also a suggestion on the construction of a slide rule for the solution, in certain cases, of the equation $U_g = U_0\,10^x$ at one setting. Application is made of the last to show the manner in which an ordinary slide rule may be used to determine the entries in an electrical wire table from the corresponding Brown and Sharpe gage number.

The remarks below are doubtless familiar to some, but as the author has nowhere seen them in print, it was thought profitable to set them down here.

Description of the Log Log Scales

The log log or LL scales of a slide rule as constructed in this country (for example, by Keuffel and Esser Co.) comprise four scales marked LL_3, LL_2, LL_1, and LL_0. In connection with the first three we also use the scale marked C and with the last the scale marked B. If the full length of the C scale is taken as unity the graduations on the various scales are: on C, $l = \log_{10} x$; on LL_3, $l = \log_{10} \log_e M$; on LL_2, $l = 1 + \log_{10} \log_e M$; on LL_1, $l = 2 + \log_{10} \log_e M$. For the remaining two scales we take the full length of the B scale as equal to 2 and the graduations are: on B, $l = \log_{10} x$; on LL_0 up to the mark .05, $l = 1 + \log_{10} \log_e M^{-1}$; and on LL_0 beyond the mark .05, $l = 3 + \log_{10} \log_e M^{-1}$. In the above l is the distance, in the unit specified, from the left index of the scale in question to the mark x or M as the case may be. The description given in this paragraph will justify the rules stated below for the use of the log log scales.

Definition of the Slide Rule Characteristic of a Number

In conformity with previous instruction given on this subject by the author, it is convenient here to define the slide rule characteristic of a number. The slide rule characteristic of a number greater than one is the number of digits to the left of the decimal, and of a number less than one it is equal to minus the number of zeros between the decimal and the first significant figure. The slide rule characteristic of a number is thus one greater than the characteristic of its logarithm to the base ten. In what follows the slide rule characteristic of a number x is designated by the symbol $S(x)$.

In order that the statements of the rules which follow, for the relations between the slide rule characteristics, may be as simple as possible, the alignments which are specified should be made, where feasible, with the left index (for example, 1 on C or B). A right index should be used only when necessary.

The Use of the LL_3, LL_2, and LL_1 Scales

The LL_3, LL_2, and LL_1 scales are used to solve the equation

$$M^x = N^y$$

when any three of the four variables involved are given. The range of the scales limits the computation to $1.01 \leqq M$, $N \leqq 22000$, $.001 \leqq x$, and $y \leqq 1000$. (See, however, the paragraph after the next.) The actual values of M and N are set on the LL scales, but only the first 3 (or 4) significant figures of x and y respectively are set on the C scale. The procedure for determining the unknown item in the above equation is:

$$\text{Align} \begin{cases} y \text{ on } C \text{ over } M \text{ on } LL_m, \\ x \text{ on } C \text{ over } N \text{ on } LL_n. \end{cases}$$

Furthermore,

I. $S(y) + n = S(x) + m$ if the slide projects to the right (or M is to the left of N);

II. $S(y) + n = S(x) + m + 1$ if the slide projects to the left (or M is to the right of N).

In each of the rules I and II all but one of the items are known and thus the unknown one is easily determined.

For the important special case $y = 1$ the rules just given may be restated in a simpler form. To solve the equation

$$M^x = N,$$

align $\begin{cases} 1 \text{ (or 10) on } C \text{ over } M \text{ on } LL_m, \\ x \text{ on } C \text{ over } N \text{ on } LL_n, \end{cases}$

where

I. $n - m + 1 = S(x)$ if the slide projects to the right (or not at all);

II. $n - m = S(x)$ if the slide projects to the left.

It is often desirable to determine the greatest and the least powers to which a given number M may be raised by the use of these scales. To do this, set 1 on C over M on LL_m. Read x on C over the right end of LL_m. Taking for x its actual value as read on C, (i.e., $1 \leq x \leq 10$), the maximum power to which M may be raised is $10^{3-m}x$ and the least power is $10^{-m}x$.

If either M or N does not lie on these scales, or if x is too large or too small (see previous paragraph), it is best to make application of the elementary laws of exponents and thus break the problem up into several others to each of which these scales are applicable. However, in some cases, where M and N are fractional, it is simpler to use the LL_0 scale.

The Use of the LL_0 Scale

The LL_0 scale covers the range $.05 \leqq M$ and $N \leqq .97$ and $.01 \leqq x \leqq 100$. The actual values of M and N are set on the LL_0 scale. Also the actual value of x is set on the B scale, $1 \leqq x \leqq 10$ on the left half and $10 \leqq x \leqq 100$ on the right half. To solve the equation

$$M^y = N,$$

$$\text{align} \begin{cases} 1 \text{ (or 100) on } B \text{ under } M \text{ on } LL_0. \\ x \text{ on } B \text{ under } N \text{ on } LL_0. \end{cases}$$

Furthermore,

I. If the slide projects to the right (or not at all),
$y = x$ if M and N are on the same side of .05;

$y = \dfrac{x}{100}$ if M and N are separated by .05.

II. If the slide projects to the left,

$y = \dfrac{x}{100}$ if M and N are on the same side of .05;

$y = x$ if M and N are separated by .05.

A Proposed Slide Rule Scale

Suppose that we are given the equation

$$U_g = U_o \, 10^{\frac{10^{s-2}}{m} g}$$

where m is a constant greater than zero and s is the slide rule characteristic of m. Let us lay off on the tongue of a slide rule a scale L, graduated for a full unit of length by the relation $l = \dfrac{10^{s-2}}{m} N$. Furthermore, let us lay off on the stock a scale D, graduated by the relation $l = \log_{10} U$. To use these scales to solve the equation

$$U_g = U_o \, 10^{\frac{10^{s-2}}{m} g}$$

first set down the relation

$$g = p \frac{m}{10^{s-2}} + N$$

where p is a positive or negative integer or zero, and

$0 \leqq N < \dfrac{m}{10^{s-2}}$.

Next

$$\text{align} \begin{cases} 0 \left(\text{or } \dfrac{m}{10^{s-2}}\right) & \text{on } L \text{ over } U_0 \text{ on } D; \\ N & \text{on } L \text{ over } U_g \text{ on } D. \end{cases}$$

Furthermore,

I. $S(U_g) = S(U_0) + p$, if the slide projects to the right (or not at all);

II. $S(U_g) = S(U_0) + p + 1$, if the slide projects to the left.

If we take $m = 1$ and $U_0 = 1$ we have the customary procedure for determining from the L and D scales of an ordinary slide rule the decimal value of a number when its logarithm to the base ten is given, and vice versa.

Application to the B. and S. Wire Table

In a Brown and Sharpe (also called A. W. G.) wire table the electrical resistance per unit length increases as the gage number goes up. However, the increase is in such a fashion that for equal lengths the ratio of the resistances of two successive gages is $\sqrt[39]{8464} = 1.2610$. Since $\sqrt[10]{10} = 1.2589$, it follows that we may use the L and D scales of the ordinary slide rule to compute, at a single setting, in the manner explained under the previous heading, the electrical resistance corresponding to a given gage number and vice versa. We may do likewise for any other quantity which varies either directly or inversely as the resistance.

For copper wire it is easy to keep in mind that, approximately, No. 10 copper wire has: a resistance of 1 ohm per

1000 feet, a cross-sectional area of 10,000 circular mils, and a weight of 31.5 pounds per 1000 feet.

In this application N is evidently the unit's digit in the B. and S. gage number, G, and since we start with No. 10 wire, p is the 10's digit of G — 10. It is to be noted that since the weight per 1000 feet and the cross-sectional area both vary inversely as the resistance, the L scale should be inserted backwards and the rules for determining the characteristic of U_g reversed, in the computation of these quantities. An alternative method is to insert the L scale forwards, but use $\overline{N} = 10 - N$ and $\overline{p} =$ minus the 10's digit of G. Here N, p, and G have the same significance as previously. The rules for the characteristic of U_g are then those given under the previous heading.

FRESHMAN TESTS ON HIGH SCHOOL ALGEBRA AND GEOMETRY

In October of the current session, tests on high school algebra and geometry were given to the freshmen in the mathematics classes in accordance with the plan inaugurated in 1928. This year four sections were not given the examination.

For lack of funds, the results of these tests have not been sent to the high schools as has been done in former years.

The plan of making separate counts for students in M. 301 (Trigonometry) and M. 302 (Analytic Geometry) has been followed again this year. Also separate records are given for those reviewing algebra in their senior high school year and for those not reviewing it.

For M. 301 and M. 302:

Number of schools	331
Number of students	866
Number of students passing	308
Number of students failing	558
Per cent of students passing	35.6
Number of students reviewing algebra in high school	227
Per cent of students reviewing algebra in high school	26.2
Per cent of students passing who reviewed algebra in high school	63.9
Per cent of students passing not reviewing algebra in high school	25.5
Per cent failing reviewing algebra in high school	36.1
Per cent failing not reviewing algebra in high school	74.5

For M. 301:

	Number	*Per Cent*
Students passing	199	30.7
Students failing	450	69.3
Students reviewing algebra in high school	161	24.8
Students not reviewing algebra in high school	448	75.2
Students passing reviewing algebra in high school	97	60.2
Students passing not reviewing algebra in high school	102	20.9
Students failing reviewing algebra in high school	64	39.8
Students failing not reviewing algebra in high school	386	79.1

For M. 302:

	Number	Per Cent
Students passing	109	50.2
Students failing	108	49.8
Students reviewing algebra in high school	66	30.4
Students not reviewing algebra in high school	151	69.6
Students passing reviewing algebra in high school	48	72.7
Students passing not reviewing algebra in high school	61	40.4
Students failing reviewing algebra in high school	18	27.3
Students failing not reviewing algebra in high school	90	59.6

The list of students making 90 or more on the test is as follows:

For M. 301:

Austin
Alexander, Woodrow 100
Hamlett, Alan 94
Kone, Marilee 100
Murphy, Myron 95
Thomas, John Fulton 94
Wells, Peter 100
Wilson, Margaret 95
Bolton
Tyler, Robert 92
Boonville (Missouri)
Kemper Military Inst.
Seay, Charles 100
Brooklyn (New York)
Mantel, Seymour G. 92
Bryan
Stephen F. Austin
Bentley, Ann 96
Corsicana
Chrisman, Pauline 98
Dallas
Highland Park
Bentley, Bonner 93
Thompson, Lucy 92
Woodrow Wilson
Taylor, Joyce 94
Sunset
Eades, Eric, Jr. 92
Eagle Pass
Riskind, Paul 100
Elizabeth (Louisiana)
Knight, Margaret 95
Exeter (New Hampshire)
Phillips Academy
Pinckney, Charles, Jr. 100
Fort Worth
Central
Van Zandt, Harris 97
Galveston
Ball
Murphy, Richard C. 90
Trask, Robert L. 90
Houston
Sam Houston
Bintliff, Charles 91
San Jacinto
Daily, Sylvia 90
Greenhill, Joe 90
Schoenman, Harriett 90
Llano
Hart, Daniel C. 94
Lubbock
Spencer, John 100
Marble Falls
Shifflette, Frances 95
Plainfield (New Jersey)
Canter, Irving 92
Shreveport (Louisiana)
Byrd
Dodd, Dulin D. 100
Sinton
Haisley, W. E., Jr. 92
Wallingford (Connecticut)
Choate Prep.
Currie, Tom 100
Washington (D.C.)
Gunston Hall
Hulen, Frances 95
Woodville
Harrison, Wilson 93

For M. 302:

Alice
Griffith, Doyle 90
Aspen (Colorado)
Kobey, Philip 100

Atlantic City
Gropper, Harry S. 99
Austin
Kershner, Virginia 100
Mitchell, T. M. 100
Ray, Floy 100
Young, Eugene 100
Brooklyn (N.Y.)
Herstone, Samuel 98
Corsicana
Hall, Warren, Jr. 94
Dallas
North Dallas
Decherd, Ben 100
Harris, John 95
Martin, J. B. 100
Nesbitt, Alice 100
Sunset
McDaniel, Vivian 100
Scaff, Alvin H. 98
Denton
Wilks, S S. 93
Fort Worth
Central
Chilton, Ernest 100
Rhea, Alice 100
Fredericksburg
Koerner, Theodore 95
Galveston
Ball
Flick, Jesse 90
Tacquard, George K. 92
Laredo
Gates, Anita 90
Lawrence (Massachusetts)
Casey, John 90
Lockhart
Kirksey, Curtis 90
Marfa
Mackles, Mark 92
O'Donnell
Hodnett, C. Truett 90
Port Jervis
Honig, Nathan 94
San Antonio
Brackenridge
Lammons, Elmo L. 90
Shreveport
Kindall, Seipert 90
Springfield (Massachusetts)
Technical School
Waite, Richard 100
Waco
Smith, A. J. 100
Weatherford
Fain, Harold 95

Number of students making

90 or more	67
80–89	66
70–79	69
60–69	106
50–59	113
Total	421

The per cent of students making above 50 is 48.5.

Attention is called especially to the effect of reviewing algebra in the high school as revealed in the percentages given above.

The schools with the best records are:

Austin	45.9% passing
Dallas	65.5% passing
Fort Worth	54.5% passing
Galveston	47.1% passing
San Antonio	54.8% passing

Notice that in several of these schools that have made such good records there has been a large per cent of students reviewing algebra in the high school.

MATHEMATICS 301 AND 302

P-Alg. means passed having reviewed algebra in high school.
P-No. means passed not having reviewed algebra in high school.
F-Alg. means failed having reviewed algebra in high school.
F-No. means failed not having reviewed algebra in high school.

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Aberdeen	1	—	—	—	1
Abilene	3	—	—	—	3
Alice	3	—	2	—	1
Alvarado	1	—	1	—	—
Alvin	1	—	—	—	1
Alvord	1	—	—	—	1
Amarillo	1	—	—	—	1
Angleton	2	—	—	—	2
Aransas Pass	2	—	—	—	2
Archer City	1	—	—	—	1
Arcadia	1	—	—	—	1
Ardmore (Okla.)	2	1	—	—	1
Arkadelphia	1	—	—	—	1
Arlington	1	—	—	—	1
Aspen (Colorado)	1	—	1	—	—
Artesia (N.M.)	2	—	1	—	1
Atlantic City (N.J.)	2	2	—	—	—
Austin	122	38	18	19	47
St. Mary's Acad'y.	2	—	—	—	2
Texas Wesleyan	3	—	1	—	2
Baird	1	—	—	—	1
Balboa (Canal Z.)	1	—	1	—	—
Baling	1	—	—	—	1
Ballinger	6	—	1	1	4
Batson	1	—	—	—	1
Batesville	1	—	—	—	1
Bardwell	1	—	—	—	1
Bartlett	1	—	—	—	1
Beaumont	4	—	1	—	3
Beeville	2	—	—	—	2
Bellville	3	—	1	—	2
Belton	5	—	—	—	5
Big Spring	3	—	1	—	2
Bogata	2	—	1	—	1
Bolton	1	1	—	—	—
Bonham	1	—	1	—	—
Boonville (Mo.)					
Kemper Mil. Inst.	2	—	2	—	—
Borger	1	1	—	—	—
Bowie	2	—	1	—	1
Brady	2	—	1	—	1
Brenham					
Blinn Memorial	1	—	—	—	1
Brooklyn (N.Y.)	1	—	1	—	—
Lane	1	—	1	—	—
Erasmus Hall	1	—	—	—	1
Brookline (Mass.)	1	—	—	—	1
Brownwood	2	—	1	—	1
Howard Payne	1	—	1	—	—
Brownsville	3	—	2	—	1

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Bryan					
Allen Academy	2	—	—	—	2
Stephen F. Austin	2	—	1	—	1
Buda	3	—	1	—	2
Burkburnett	1	—	—	—	1
Burnet	1	—	1	—	—
Caldwell	2	—	—	—	2
Cameron	4	—	—	—	4
Canon City (Colo.)					
Abby High	1	—	—	—	1
Carmi (Illinois)	1	—	—	—	1
Charleston (S.C.)	1	—	—	—	1
Chattanooga					
(Tenn.)	1	—	1	—	—
Chillicothe	1	—	—	—	1
Childress	1	—	—	—	1
Cisco	1	—	—	—	1
Clarksville	1	—	—	—	1
Cleburne	3	—	1	—	2
Cleveland (Ohio)	1	1	—	—	—
Coleman	4	—	1	1	2
Coolidge	3	—	—	—	3
Comanche	1	—	—	—	1
Corpus Christi	6	—	1	1	4
Corsicana	8	3	2	—	3
I. O. O. F.	1	—	—	—	1
Crockett	3	—	1	—	2
Crosby	1	—	—	—	1
Crystal City	1	—	—	—	1
Dallas					
Forest Avenue	6	4	1	1	—
Highland Park	13	3	4	—	6
North Dallas	12	8	—	1	3
Oak Cliff	9	4	1	3	1
St. Mary's	1	1	—	—	—
Sunset	3	3	—	—	—
Terrill Prep.	8	—	2	—	6
Woodrow Wilson	13	6	3	3	1
Tech.	2	1	—	—	1
Del Rio	5	—	1	—	4
Denison	4	—	—	2	2
Des Moines (Iowa)	1	—	1	—	—
Denton	3	—	1	—	2
Denver (Colorado)					
East	1	—	—	1	—
Devine	1	—	—	—	1
Dilley	1	—	—	—	1
Donna	1	—	—	—	1
Drumwright (Okla.)	1	—	—	—	1
Dublin	2	—	—	—	2
Dumas	1	—	—	—	1
Eagle Lake	2	—	1	—	1
Eastland	3	—	—	—	3
Edna (Ind.)	1	—	—	—	1
Edna	3	—	—	—	3
El Campo	1	—	1	—	—
Elizabeth (N.J.)	1	1	—	—	—
Elizabeth (La.)	1	1	—	—	—

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Elgin	4	2			2
El Paso	5	2			3
Ennis	2				2
Exeter (N.H.)					
Phillips Academy	1		1		
Fabens	1				1
Falconer (N.Y.)	1			1	
Flatonia	1				1
Florence	1				1
Floresville	2		1		1
Frankston	1		1		
Fredericksburg	5	3			2
Fort Smith (Ark.)	1				1
Fort Stockton	2		1		1
Fort Worth					
Central	14	7	3	1	3
Masonic Home	1				1
North Side	1		1		
Polytechnic	3		1		2
W. C. Stripling	4			1	3
Gainesville	2			1	1
Galveston					
Ball	17	6	2	5	4
Gatesville	2				2
Georgetown	1		1		
Giddings	1				1
Gilmer	2				2
Goldthwaite	2				2
Goose Creek					
Robert E. Lee	7		2	1	4
Graham	2				2
Granger	2				2
Grand Saline	1				1
Grapeland	2				2
Greenville	2			1	1
Greenwood	1				1
Gulf	1				1
Hamilton	1				1
Harlingen	5				5
Harper	1				1
Henderson	3		1		2
Hearne	1				1
Henrietta	2				2
Hillsboro	1				1
Holly Springs					
(Miss.)	1		1		
Hollis	1			1	
Hondo	2				2
Houston					
Jefferson Davis	1				1
John H. Reagan	5	1		1	3
Sam Houston	7	2	2	1	2
San Jacinto	31	9	3	11	8
South End Jr.	1				1
St. Agnes Acad.	1				1
Hull-Daisette	2			1	1
Huntsville	1		1		
Hutto	2				2

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Iowa Park	1	----	----	----	1
Jacksboro	1	----	----	----	1
Jamaica (N.Y.)	1	----	1	----	----
Johnson City	1	----	----	----	1
Jourdanton	2	----	----	----	2
Junction	1	----	----	----	1
Kaufman	1	----	----	----	1
Kerrville Schreiner Inst.	1	----	----	----	1
Kenedy	2	----	1	----	1
Kilgore	1	----	----	----	1
Kingsville	1	----	----	----	1
Kirbyville	2	----	1	----	1
Kirwin	2	----	----	----	2
Kokomo (Indiana)	1	1	----	----	----
Koscinsko (Miss.)	1	----	1	----	----
Kyle	1	----	1	----	----
Lake Forest (Ill.)	1	----	1	----	----
Lampasas	1	----	----	----	1
Lancaster	1	----	----	----	1
Laredo	7	----	2	----	5
Lawrence (Mass.)	1	----	1	----	-
Leander	1	----	----	----	1
Lexington	1	----	----	----	1
Liberty	3	----	2	----	1
Liberty Hill	1	----	----	----	1
Littlefield	2	----	----	----	2
Little Rock (Ark.)	1	----	----	----	1
Livingston	2	----	1	----	1
Lockhart	1	----	1	----	----
Llano	6	----	2	----	4
Lometa	2	----	----	----	2
Longview	2	----	1	----	1
Lubbock	4	1	----	----	3
Lufkin	2	----	----	----	2
Luling	3	----	1	----	2
Lyford	1	----	----	----	1
Madisonville (Ky.)	1	----	----	----	1
Mansfield (Mo.)	1	----	----	----	1
Marble Falls	3	2	----	----	1
Marianna (Ark.)	1	----	----	----	1
Marfa	3	1	1	----	1
Marlin	3	----	1	----	2
Marshall	2	----	----	----	2
Mart	2	----	1	----	1
Mathis	1	----	----	----	1
McAllen	3	1	----	----	2
McCamey	1	----	----	----	1
McGregor	2	----	1	----	1
McKinley	1	----	1	----	-
McKinney	2	----	----	----	2
Memphis Tech.	1	----	----	----	1
Menard	2	----	----	----	2
Meridian	1	----	----	----	1
Merkel	2	----	1	----	1
Mexia	2	----	----	----	2

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Mexico					
D. F. Am. School	1	----	----	----	1
Middleboro (Mass.)					
Memorial	2	----	1	----	1
Mineral Wells	1	----	----	----	1
Mission	2	----	1	----	1
Montgomery	1	----	----	----	1
Monroe (La.)	1	----	----	----	1
Mount Pleasant	1	----	----	----	1
Mount Vernon	1	----	----	----	1
Nacogdoches	2	----	----	----	2
Navasota	4	----	----	----	4
New Braunfels	6	2	2	1	1
New Haven (Conn.)					
Millhouse	1	----	----	----	1
New York (N.Y.)					
DeWitt Clinton	1	----	----	----	1
Textile School	1	----	----	----	1
Theo. Roosevelt	1	1	----	----	----
Townsend Harris Hall	1	----	1	----	----
Noblesville	1	----	1	----	----
Nocona	1	----	----	----	1
Oklahoma City (Okla.)					
Central	1	----	----	----	1
O'Donnell	1	----	1	----	----
Olney	1	----	----	----	1
Orange	2	----	1	----	1
Oskaloosa (Iowa)	1	----	----	----	1
Ouachita Parish (La.)	2	2	----	----	----
Palestine	5	----	1	----	4
Panhandle	2	----	----	----	2
Paterson (N.J.)	1	----	----	1	----
Pearsall	1	1	----	----	----
Philadelphia (Pa.)					
Haverford Town.	1	----	----	----	1
Pflugerville	1	----	----	1	----
Plainfield (N.J.)	1	1	----	----	----
Plainview	3	----	----	----	3
Pleasanton	1	----	----	----	1
Port Arthur	6	2	----	----	4
Port Lavaca	2	----	2	----	----
Port Jervis	1	----	1	----	----
Portale (N.M.)	1	----	----	----	1
Poteet	1	----	----	----	1
Pottsville	1	----	----	----	1
Prairie Lea	1	----	----	----	1
Pharr-San Juan-Alamo	1	----	----	1	----
Quanah	1	----	----	----	1
Ralls	1	----	----	----	1
Richland Springs	1	----	----	----	1
Richmond	1	----	----	----	1
Rising Star	1	----	----	----	1
Robstown	1	----	----	----	1
Rosebud	1	----	----	----	1
Rosenberg	2	----	1	----	1

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Roswell (N.M.)					
N. M. Mil. Inst.	1	----	----	----	1
Rusk	1	----	----	----	1
Salamanca (N.Y.)	1	----	----	----	1
San Angelo	5	----	2	----	3
San Antonio					
Alamo Heights	4	1	----	3	----
Brackenridge	6	----	2	----	4
Main Avenue	9	2	3	1	3
Incarnate Word	1	----	----	1	----
Peacock's	1	----	----	----	1
St. Mary's Hall	4	----	2	2	----
Thomas Jefferson	12	6	2	2	2
Texas Mil. Inst.	3	1	1	----	1
San Benito	1	----	----	----	1
San Marcos	3	----	1	----	2
Baptist Academy	5	----	----	1	4
San Saba	1	----	----	----	1
Sanderson	2	----	1	----	1
Sayle (La.)	1	----	----	----	1
Schulenberg	1	----	----	----	1
Senatobia (Miss.)	1	----	1	----	----
Sewanee Mil. Acad.	1	----	----	----	1
Shamrock	1	----	----	----	1
Sharyland	1	1	----	----	----
Sherman	1	----	----	----	1
Shiner	4	----	1	----	3
Shreveport (La.)					
Byrd High	3	2	----	1	----
Siloam Spring	1	1	----	----	----
Sinton	2	1	----	----	1
Smithville	3	----	----	----	3
Sonora	1	----	----	----	1
Sour Lake	1	----	----	----	1
Springfield (Mass.)					
Technical School	2	1	----	1	----
St. Mary's (Kan.)	1	----	1	----	----
St. Louis (Mo.)	1	----	----	----	1
Mary Inst.	1	----	----	----	1
Sturgis	1	----	1	----	-
Sugar Land	3	----	1	----	2
Sulphur Springs	1	----	----	----	1
Taft	2	----	1	----	1
Tarragut (Iowa)	1	----	----	----	1
Taylor	4	----	----	3	1
Temple	2	----	----	1	1
Terrill	1	----	1	----	----
Texarkana	5	----	1	----	4
Texas City	1	----	----	----	1
Timmins (Ontario)	1	----	1	----	----
Thorndale	1	----	----	----	1
Tolar	1	----	----	----	1
Tulsa (Okla.)	1	----	1	----	----
Turtle Creek (Pa.)	1	----	1	----	-
Tyler	4	----	----	2	2
Victoria	3	----	----	----	3
Waco	12	1	3	1	7

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Wallingford (Conn.) Choate Prep.	1	1	----	----	----
Wanocoma (Iowa)	1	----	----	----	1
Washington (D.C.)	1	1	----	----	----
Wansa (Neb.)	1	----	----	----	1
Waxahachie	1	1	----	----	----
Weatherford	3	1	2	----	----
Webb (Miss.)	1	1	----	----	----
West	3	----	----	----	3
Wharton	1	----	----	----	1
Whitewright	1	----	----	----	1
Wichita Falls	6	----	2	----	4
Wills Point	3	----	----	----	3
Winnetka (Ill.) New Trier	1	----	----	----	1
Winnsboro	1	----	----	----	1
Winters	2	----	----	----	2
Winthrop (Mass.)	1	1	----	----	----
Weslaco	1	----	----	----	1
Woodstock Community	1	----	1	----	----
Woodville	2	----	1	----	1
Wortham	2	----	----	----	2
Yoakum	3	----	----	----	3
TOTAL	866	149	159	82	476

MATHEMATICS 301

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Aberdeen	1	----	----	----	1
Abilene	2	----	----	----	2
Alice	2	----	1	----	1
Alvarado	1	----	1	----	----
Alvin	1	----	----	----	1
Alvord	1	----	----	----	1
Amarillo	1	----	----	----	1
Angleton	2	----	----	----	2
Aransas Pass	2	----	----	----	2
Arcadia	1	----	----	----	1
Ardmore (Okla.)	1	1	----	----	----
Arlington	1	----	----	----	1
Artesia (N.M.)	1	----	----	----	1
Atlantic City (N.J.)	1	1	----	----	----
Austin	83	21	13	12	37
St. Mary's Acad.	2	----	----	----	2
Texas Wesleyan	3	----	1	----	2
Baling	1	----	----	----	1
Ballinger	5	----	----	1	4
Batson	1	----	----	----	1
Batesville	1	----	----	----	1
Bardwell	1	----	----	----	1
Bartlett	1	----	----	----	1
Beaumont	2	----	----	----	2
Beeville	2	----	----	----	2
Bellville	2	----	----	----	2
Belton	4	----	----	----	4

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Big Spring	2	—	—	—	2
Bolton	1	1	—	—	—
Bonham	1	—	1	—	—
Boonville (Mo.)					
Kemper Mil. Inst.	2	—	2	—	—
Borger	1	1	—	—	—
Bowie	1	—	—	—	1
Brady	2	—	1	—	1
Brenham					
Blinn Memorial	1	—	—	—	1
Brooklyn (N.Y.)					
Lane	1	—	1	—	—
Erasmus Hall	1	—	—	—	1
Brookline (Mass.)	1	—	—	—	1
Brownwood	2	—	1	—	1
Brownsville	2	—	1	—	1
Bryan					
Stephen F. Austin	2	—	1	—	1
Allen Academy	1	—	—	—	1
Buda	3	—	1	—	2
Burkburnett	1	—	—	—	1
Cameron					
Yoe	4	—	—	—	4
Carmi (Ill.)	1	—	—	—	1
Charleston (S.C.)	1	—	—	—	1
Chillicothe	1	—	—	—	1
Childress	1	—	—	—	1
Cisco	1	—	—	—	1
Clarksville	1	—	—	—	1
Cleburne	3	—	1	—	2
Cleveland (Ohio)	1	1	—	—	—
Coleman	4	—	1	1	2
Coolidge	2	—	—	—	2
Corpus Christi	5	—	1	1	3
Corsicana	6	3	—	—	3
I. O. O. F.	1	—	—	—	1
Crockett	3	—	1	—	2
Crosby	1	—	—	—	1
Crystal City	1	—	—	—	1
Dallas					
Forest Avenue	4	2	1	1	—
Highland Park	11	3	3	—	5
North Dallas	7	3	—	1	3
Oak Cliff	5	2	1	1	1
St. Mary's Acad.	1	1	—	—	—
Sunset	1	1	—	—	—
Terrill Prep.	4	—	—	—	4
Tech.	2	1	—	—	1
Woodrow Wilson	9	4	1	3	1
Del Rio	5	—	1	—	4
Denison	4	—	—	2	2
Denton	2	—	—	—	2
Denver (Colo.)					
East	1	—	—	1	—
Devine	1	—	—	—	1
Dilley	1	—	—	—	1
Donna	1	—	—	—	1
Drumwright (Okla.)	1	—	—	—	1

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Dublin	2	----	----	----	2
Eagle Lake	2	----	1	----	1
Eastland	3	----	----	----	3
Edna (Ind.)	1	----	----	----	1
Edna	2	----	----	----	2
El Campo	1	----	1	----	----
Elizabeth (N.J.)	1	1	----	----	----
Elizabeth (La.)	1	1	----	----	----
Elgin	4	----	2	----	2
El Paso	5	----	2	----	3
Ennis	2	----	----	----	2
Exeter (N.H.)					
Phillips Academy	1	----	1	----	----
Fabens	1	----	----	----	1
Falconer (N.Y.)	1	----	----	1	----
Floresville	1	----	----	----	1
Flatonia	1	----	----	----	1
Frankston	1	----	1	----	----
Fredericksburg	3	1	----	----	2
Fort Smith (Ark.)	1	----	----	----	1
Fort Stockton	1	----	----	----	1
Fort Worth					
Central	8	5	1	1	1
Masonic Home	1	----	----	----	1
North Side	1	----	1	----	----
Polytechnic	3	----	1	----	2
W. C. Stripling	2	----	----	1	1
Galveston					
Ball	12	4	2	4	2
Gatesville	2	----	----	----	2
Gilmer	2	----	----	----	2
Goose Creek					
Robert E. Lee	6	----	2	----	4
Graham	1	----	----	----	1
Granger	1	----	----	----	1
Grapeland	1	----	----	----	1
Greenville	2	----	----	1	1
Greenwood	1	----	----	----	1
Gulf	1	----	----	----	1
Hamilton	1	----	----	----	1
Harlingen	5	----	----	----	5
Harper	1	----	----	----	1
Henderson	2	----	1	----	1
Henrietta	2	----	----	----	2
Hillsboro	1	----	----	----	1
Holly Springs					
(Miss.)	1	----	1	----	----
Hollis	1	----	----	1	----
Hondo	2	----	----	----	2
Houston					
John H. Reagan	3	----	----	1	2
Sam Houston	6	2	2	----	2
San Jacinto	24	8	2	9	5
South End Jr.	1	----	----	----	1
Hull-Daisette	1	----	----	1	----
Huntsville	1	----	1	----	----
Hutto	1	----	----	----	1
Jacksboro	1	----	----	----	1

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Johnson City	1	—	—	—	1
Jourdanton	2	—	—	—	2
Junction	1	—	—	—	1
Kaufman	1	—	—	—	1
Kerrville					
Schreiner Inst.	1	—	—	—	1
Kilgore	1	—	—	—	1
Kingsville	1	—	—	—	1
Kirbyville	2	—	1	—	1
Kirwin	2	—	—	—	2
Kokomo (Indiana)	1	1	—	—	—
Koscinsko (Miss.)	1	—	1	—	—
Kyle	1	—	1	—	—
Lake Forest (Ill.)	1	—	1	—	—
Llano	5	—	1	—	4
Lampasas	1	—	—	—	1
Lancaster	1	—	—	—	1
Laredo	5	—	1	—	4
Leander	1	—	—	—	1
Lexington	1	—	—	—	1
Liberty Hill	1	—	—	—	1
Liberty	3	—	2	—	1
Littlefield	2	—	—	—	2
Little Rock (Ark.)	1	—	—	—	1
Livingston	1	—	—	—	1
Lometa	2	—	—	—	2
Longview	1	—	—	—	1
Lubbock	3	1	—	—	2
Lufkin	2	—	—	—	2
Luling	3	—	1	—	2
Lyford	1	—	—	—	1
Madisonville (Ky.)	1	—	—	—	1
Mansfield (Mo.)	1	—	—	—	1
Marble Falls	2	1	—	—	1
Marianna (Ark.)	1	—	—	—	1
Marfa	2	—	1	—	1
Marlin	3	—	1	—	2
Marshall	2	—	—	—	2
Mart	1	—	—	—	1
McAllen	2	—	—	—	2
McCamey	1	—	—	—	1
McGregor	2	—	1	—	1
McKinley	1	—	1	—	—
McKinney	2	—	—	—	2
Memphis					
Tech.	1	—	—	—	1
Menard	2	—	—	—	2
Meridian	1	—	—	—	1
Merkel	2	—	1	—	1
Mexia	2	—	—	—	2
Mexico					
D. F. Amer. School	1	—	—	—	1
Middleboro (Mass.)					
Memorial	2	—	1	—	1
Mineral Wells	1	—	—	—	1
Mission	2	—	1	—	1
Montgomery	1	—	—	—	1

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Monroe (La.)	1	----	----	----	1
Mt. Vernon	1	----	----	----	1
Nacogdoches	2	----	----	----	2
Navasota	4	----	----	----	4
New Braunfels	6	2	2	1	1
New Haven (Conn.)					
Millhouse	1	----	----	----	1
New York (N. Y.)					
DeWitt Clinton	1	----	----	----	1
Townsend Harris Hall	1	----	1	----	----
Theo. Roosevelt	1	1	----	----	--
Nocona	1	----	----	----	1
Oklahoma City, Okla.					
Central	1	----	----	----	1
Olney	1	----	----	----	1
Ouachita Par. (La.)	2	2	----	----	----
Orange	1	----	----	----	1
Oskaloosa (Ia.)	1	----	----	----	1
Palestine	5	----	1	----	4
Panhandle	1	----	----	----	1
Paterson (N. J.)	1	----	----	1	----
Pearsall	1	1	----	----	----
Philadelphia (Penn.)					
Haverford Township	1	----	----	----	1
Plainfield (N. J.)	1	1	----	----	----
Plainview	2	----	----	----	2
Pleasanton	1	----	----	----	1
Pt. Arthur	5	2	----	----	3
Pt. Lavaca	2	----	2	----	----
Portale (N.M.)	1	----	----	----	1
Pottsville	1	----	----	----	1
Prairie Lea	1	----	----	----	1
Richmond	1	----	----	----	1
Rising Star	1	----	----	----	1
Robstown	1	----	----	----	1
Rosebud	1	----	----	----	1
Rosenberg	2	----	1	----	1
Roswell (N.M.)					
N. M. Mil. Inst.	1	----	----	----	1
Rusk	1	----	----	----	1
San Angelo	4	----	1	----	3
San Antonio					
Alamo Heights	4	1	----	3	----
Brackenridge	4	----	1	----	3
Main Avenue	6	1	2	1	2
Thomas Jefferson	10	4	2	2	2
Incarnate Word	1	----	----	1	----
St. Mary's Hall	3	----	1	2	----
Tex. Military Inst.	2	1	1	----	----
Sanderson	1	----	----	----	1
San Marcos	2	----	1	----	1
Baptist Academy	5	----	----	1	4
San Saba	1	----	----	----	1
Sayle (La.)	1	----	----	----	1
Senatobia (Miss.)	1	----	1	----	----

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Sewanee Mil. Acad.	1	----	----	----	1
Shamrock	1	----	----	----	1
Sharyland	1	1	----	----	----
Sherman	1	----	----	----	1
Shiner	4	----	1	----	3
Shreveport (La.)	2	1	----	1	----
SiloamSpring (Ark.)	1	1	----	----	----
Sinton	2	1	----	----	1
Smithville	2	----	----	----	2
Sonora	1	----	----	----	1
Springfield (Mass.) Technical School	1	----	----	1	----
St. Louis (Mo.)	1	----	----	----	1
Mary Inst.	1	----	----	----	1
Sturgis	1	----	1	----	----
Sugar Land	1	----	----	----	1
Sulphur Springs	1	----	----	----	1
Taft	1	----	1	----	----
Tarragut (Ia.)	1	----	----	----	1
Taylor	4	----	----	3	1
Temple	2	----	----	1	1
Texarkana	3	----	1	----	2
Texas City	1	----	----	----	1
Timmins (Ont.)	1	----	1	----	----
Tolar	1	----	----	----	1
Tulsa (Okla.)	1	----	1	----	----
Tyler	3	----	----	1	2
Victoria	2	----	----	----	2
Waco	9	1	1	1	6
Wallingford (Conn.) Choate Prep.	1	1	----	----	----
Wanocoma (Ia.)	1	----	----	----	1
Washington (D.C.)	1	1	----	----	----
Wansa (Neb.)	1	----	----	----	1
Waxahachie	1	1	----	----	----
Weatherford	1	----	1	----	----
Webb (Miss.)	1	1	----	----	----
West	2	----	----	----	2
Wharton	1	----	----	----	1
Whitewright	1	----	----	----	1
Wichita Falls	3	----	1	----	2
Wills Point	3	----	----	----	3
Winnetka (Ill.) New Trier	1	----	----	----	1
Winnsboro	1	----	----	----	1
Winters	2	----	----	----	2
Winthrop (Mass.)	1	1	----	----	----
Woodstock Community	1	----	1	----	----
Woodville	2	----	1	----	1
Wortham	2	----	----	----	2
Yoakum	3	----	----	----	3
TOTAL	649	97	102	64	386

MATHEMATICS 302

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Abilene	1	----	----	----	1
Austin	39	17	5	7	10
Alice	1	----	1	----	----
Artesia (N. M.)	1	----	1	----	----
Archer City	1	----	----	----	1
Ardmore	1	----	----	----	1
Arkadelphia	1	----	----	----	1
Aspen (Col.)	1	----	1	----	----
Atlantic City	1	1	----	----	----
Baird	1	----	----	----	1
Balboa (Canal Zone)	1	----	1	----	----
Ballinger	1	----	1	----	----
Beaumont	2	----	1	----	1
Bellville	1	----	1	----	----
Belton	1	----	----	----	1
Big Spring	1	----	1	----	----
Bogata	2	----	1	----	1
Bowie	1	----	1	----	----
Brooklyn (N. Y.)	1	----	1	----	----
Brownsville	1	----	1	----	----
Brownwood					
Howard Payne	1	----	1	----	----
Bryan					
Allen Academy	1	----	----	----	1
Burnet	1	----	1	----	----
Caldwell	2	----	----	----	2
Canon City (Colo.)					
Abby High	1	----	----	----	1
Chattanooga (Tenn.)	1	----	1	----	----
Coolidge	1	----	----	----	1
Comanche	1	----	----	----	1
Corpus Christi	1	----	----	----	1
Corsicana	2	----	2	----	----
Dallas					
Forest Avenue	2	2	----	----	----
Highland Park	2	----	1	----	1
North Dallas	5	5	----	----	----
Oak Cliff	4	2	----	2	----
Sunset	2	2	----	----	----
Woodrow Wilson	4	2	2	----	----
Terrill Prep.	4	----	2	----	2
Denton	1	----	1	----	----
Des Moines (Ia)	1	----	1	----	----
Dumas	1	----	----	----	1
Edna	1	----	----	----	1
Florence	1	----	----	----	1
Floresville	1	----	1	----	----
Fredericksburg	2	2	----	----	----
Ft. Stockton	1	----	1	----	----
Ft. Worth					
Central	6	2	2	----	2
Stripling	2	----	----	----	2
Gainesville	2	----	----	1	1
Galveston					
Ball	5	2	----	1	2

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Georgetown	1	----	1	----	----
Giddings	1	----	----	----	1
Goldthwaite	2	----	----	----	2
Goose Creek	1	----	----	1	----
Graham	1	----	----	----	1
Granger	1	----	----	----	1
Grand Saline	1	----	----	----	1
Grapeland	1	----	----	----	1
Henderson	1	----	----	----	1
Hearne	1	----	----	----	1
Houston					
San Jacinto	7	1	1	2	3
Jefferson Davis	1	----	----	----	1
John H. Reagan	2	1	----	----	1
Sam Houston	1	----	----	1	----
St. Agnes Acad.	1	----	----	----	1
Hull-Daisette	1	----	----	----	1
Hutto	1	----	----	----	1
Iowa Park	1	----	----	----	1
Jamaica (N. Y.)	1	----	1	----	----
Kenedy	2	----	1	----	1
Llano	1	----	1	----	----
Laredo	2	----	1	----	1
Lawrence (Mass.)	1	----	1	----	----
Livingston	1	----	1	----	----
Lockhart	1	----	1	----	----
Longview	1	----	1	----	----
Lubbock	1	----	----	----	1
Marble Falls	1	1	----	----	----
Marfa	1	1	----	----	----
Mart	1	----	1	----	----
Mathis	1	----	----	----	1
McAllen	1	1	----	----	----
Mt. Pleasant	1	----	----	----	1
New, York (N. Y.)					
Textile School	1	----	----	----	1
Noblesville	1	----	1	----	----
O'Donnell	1	----	1	----	----
Orange	1	----	1	----	----
Panhandle	1	----	----	----	1
Pflugerville	1	----	----	1	----
Pharr					
San Juan Alamo	1	----	----	1	----
Plainview	1	----	----	----	1
Pt. Arthur	1	----	----	----	1
Pt. Jervis	1	----	1	----	----
Poteet	1	----	----	----	1
Quanah	1	----	----	----	1
Ralls	1	----	----	----	1
Richland Springs	1	----	----	----	1
Salamanca (N. Y.)	1	----	----	----	1
San Angelo	1	----	1	----	----
San Antonio					
Brackenridge	2	----	1	----	1
Main Avenue	3	1	1	----	1
Thomas Jefferson	2	2	----	----	----
St. Mary's Hall	1	----	1	----	----

School	*No.*	*P-Alg.*	*P-No.*	*F-Alg.*	*F-No.*
Texas Mil. Inst.	1	----	----	----	1
Peacock's	1	----	----	----	1
San Benito	1	----	----	----	1
San Marcos	1	----	----	----	1
Sanderson	1	----	1	----	----
Schulenberg	1	----	----	----	1
Shreveport (La.) Byrd High	1	1	----	----	----
Smithville	1	----	----	----	1
Sour Lake	1	----	----	----	1
Springfield (Mass.) Tech. School	1	1	----	----	----
St. Mary's (Kan.)	1	----	1	----	----
Sugar Land	2	----	1	----	1
Taft	1	----	----	----	1
Terrill	1	----	1	----	----
Texarkana	2	----	----	----	2
Thorndale	1	----	----	----	1
Turtle Creek (Pa.)	1	----	1	----	----
Tyler	1	----	----	1	----
Victoria	1	----	----	----	1
Waco	3	----	2	----	1
Weatherford	2	1	1	----	----
Weslaco	1	----	----	----	1
West	1	----	----	----	1
Wichita Falls	3	----	1	----	2
TOTAL	217	48	61	18	90